# CHAPTER 3

# NUMERICAL EXAMPLES

## 3.1 DYNAMIC ADJUSTMENT OF AGE DISTRIBUTION IN KNOWLEDGE WORKER MANAGEMENT

The first problem is reconsidered here again. The HRM policy for an age distribution is stated as follows. For a given age distribution at present year $P_0$ = $\{E_{A1}(t_0),\ldots,E_{Aj}(t_0),\ldots,E_{ANAge}(t_0)\}$, it is desired that the age distribution $P_{NYear}$ = $\{E_{A1}(t_{NYear}),\ldots, E_{Aj}(t_{NYear}),\ldots,E_{ANAge}(t_{NYear})\}$ in the next *NYear* years after the present year $t_0$ be close to the desired age distribution $P_D = \{E_{A1}{}^D,\ldots, E_{Aj}{}^D,\ldots,E_{ANAge}{}^D\}$ as much as possible. The adjustment of the age distributions $P_0$ to $P_{NYear}$ is achieved via the consecutive adjustment in the number of employees at various ages. Accordingly, a mathematical expression which represents such a procedure can be given as

$$E_{A_j}(t_i) = E_{A_{j-1}}(t_{i-1}) + \delta_{A_j}(t_i) \quad ; \; i = 1,\ldots,NYear \text{ and } j = 1,\ldots,NAge \quad (3.1)$$

$$E_{A_0}(t_i) = 0 \quad ; \; \forall i \quad (3.2)$$

$$A_j - A_{j-1} = 1 \quad ; \; \forall j \quad (3.3)$$

and

$$t_i - t_{i-1} = 1 \quad ; \; \forall i \quad (3.4)$$

, where $E_{Aj}(t_i) \in \Re$ and $\delta_{Aj}(t_i) \in \Re$. Note that (3.3) says the difference between the consecutive age group is equal to 1 year old. The number of employees $E_{Aj}(t_i)$ is taken as percentage of total number of employees, i.e.

$$\sum_{j=1}^{NAge} E_{Aj}(t_i) = 100 \quad ; \forall i \quad (3.5)$$

This adjustment has to be decided and implemented by the HRM department and is the subject of the study in this paper. When $\delta_{Aj}(t_i)$ equals to 0's for all $A_j$'s and $t_i$'s, (1) is just an age evolution for every passing year. Consequently, (3.1) describes both the dynamics of age evolution and the process of the age-distribution adjustment. Equation (3.1) may be extended to the case of resignation. However, it is beyond the scope of this study.

According to the HRM policy for an age distribution above, the following optimization problem is formulated:

$$\underset{\delta_{Aj}(t_i)}{Min}\ ERR = \sum_{j=1}^{NAge}\left(E_{Aj}^{D} - E_{Aj}\left(t_{NYear}\right)\right)^2 \quad ; i = 1,\ldots,NYear \text{ and } j = 1,\ldots,NAge \qquad (3.6.1)$$

, or

$$\underset{\delta_{Aj}(t_i)}{Min}\ ERR = \sum_{j=1}^{NAge}\left(E_{Aj}^{D} - E_{Aj}\left(\delta_{Aj}\left(t_i\right)\right)\right)^2 \quad ; i = 1,\ldots,NYear \text{ and } j = 1,\ldots,NAge \qquad (3.6.2)$$

, subject to

$$E_{Aj}\left(t_i\right) \geq 0 \quad ; \forall i \text{ and } \forall j \qquad (3.7)$$

and

$$\sum_{j=1}^{NAge} E_{Aj}\left(t_i\right) = 100 \quad ; \forall i \qquad (3.8)$$

where

*NAge* : The total number of age groups

*Nyear* : The year that the adjustment in the age distribution is expected to meet the desired age distribution

$P_0$ : The age distribution at present year

$P_{NYear}$ : The age distribution at the *NYear*th year after the present year

$P_D$ : The desired age distribution

$A_j$ : The *j*th age group

$t_i$ : The *i*th year

$E_{Aj}(t_i)$ : The number of employees in the age group $A_j$ at time $t_i$

$E_{Aj}^{D}$ : The desired number of employees in the age group $A_j$

$\delta_{Aj}(t_i)$ : The adjustment magnitude of the number of employees in the age group $A_j$ at time $t_i$

$\Re$ : The set of real numbers

$ERR$ : The total discrepancy between $P_{NYear}$ and $P_D$

$\delta_{Aj}(t_i)$, $i = 1,\ldots,NYear$ and $j = 1,\ldots,NAge$, are the variables to be optimally determined. A set of inequality constraints (3.7) are required for the non-negativity of the number of employees at all times. The 100-percentage criteria of the number of employees at all times, as given previously by (3.3), are preserved by a set of equality constraints (3.8). The optimization problem considered here involves the dynamics of a system subjected to multiple constraints.

The fitness function of a chromosome $F(\Delta)$ is defined as

$$F(\Delta) = \frac{1}{O(\Delta)} \quad ; \Delta = \{\delta_{Aj}(t_i) \mid i = 1,\ldots,NYear \text{ and } j = 1,\ldots,NAge\} \qquad (3.9)$$

, in which $O(\Delta)$ is defined as

$$O(\Delta) = \begin{cases} ERR(\Delta) & ; \Delta \text{ is feasible} \\ ERR(\Delta) + \sum_{j=1}^{NCon} c_k v_k(\Delta) & ; \Delta \text{ is infeasible} \end{cases} \qquad (3.10)$$

$v_k(\Delta)$ : The violation magnitude of the $k$th constraint

$<v_k(\Delta)>$ : The average of $v_k(\Delta)$ over the population

$c_k$ : The penalty parameter for the $k$th constraint defined at each generation

$NCon$ : The total number of constraints

$F(\Delta)$ : The fitness function

$\varepsilon$ : The tolerance for the 100-percent criteria

Following the adaptive penalty GA described previously, the penalty factor $c_k$ is given by

$$c_k = \left|max\left(ERR_{inf}(\Delta)\right)\right| \frac{< v_k(\Delta) >}{\sum_{l=1}^{NCon}\left[< v_l(\Delta) >\right]^2} \qquad (3.11)$$

The equality constraint (3.8) is modified to be an inequality constraint

$$\frac{\left|\sum_{j=1}^{NAge} E_{Aj}(t_i) - 100\right|}{100} \le \varepsilon \qquad ; \forall i \qquad (3.12)$$

,where $max(ERR_{inf}\ (\Delta))$ is the maximum of the objective function values for the current population in the infeasible region. The tolerance $\varepsilon$ can be arbitrarily set but is normally a small value.

Three cases of study are considered in this paper. In all cases, the range of age groups includes the age from $A_1$ = 25 years old to $A_{35}$ = 59 years old. This implies that the youngest age that will be taken into the organization is 25 and the employee retires from the organization after 59 years old. *NYear* is equal to 5. In other words, the desired age distribution is target at the next 5 years after this present year. The present age distribution $P_0$ and the desired age distribution $P_5$ are shown in Figure 1. The tolerance for the 100-percent criteria $\varepsilon$ is set to be 0.01.

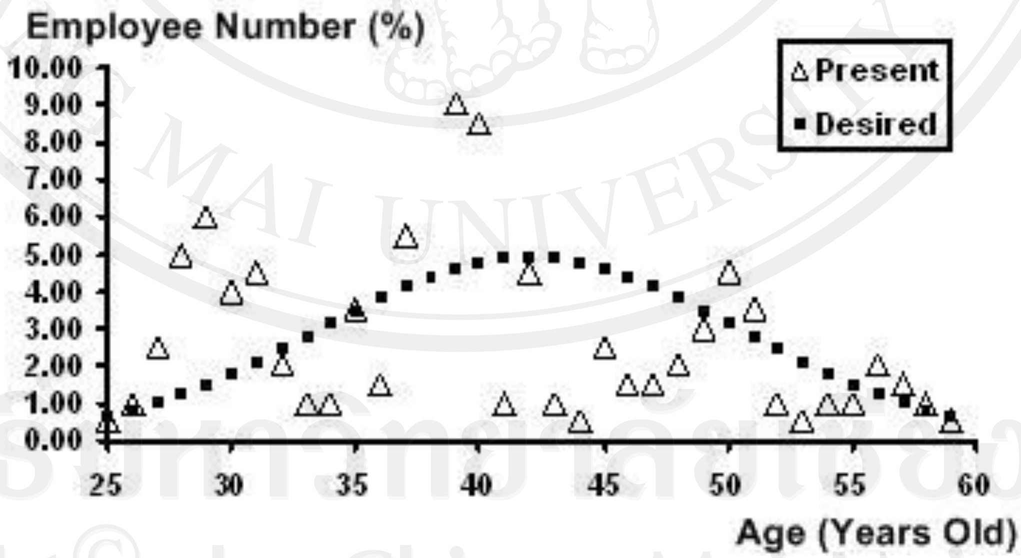



Figure 3.1. The present age distribution $P_0$ and the desired age distribution $P_5$ at 5 years.

The first case is considered as follows. The adjustment of the number of employees is performed specifically at the age groups $A_1$ = 25, $A_{11}$ = 35, $A_{16}$ = 40, and $A_{26}$ = 50. The adjustments in these age groups are time-invariant. That is, $\delta_{A1}(t_i) = \delta_{A1}$, $\delta_{A11}(t_i) = \delta_{A11}$, $\delta_{A16}(t_i) = \delta_{A16}$, and $\delta_{A26}(t_i) = \delta_{A26}$, whereas the other adjustments are equal to null. Total number of constraints *NCon* is equal to 30.

Ten simulations of GA, each of which evolves for 500 generations, are performed. The population size is 100. The best result from all simulations is taken as the adjustment magnitudes (see Figure 3.2) and is used to compute the age distribution at *NYear* = 5 (see Figure 3.3). Figure 3.4 shows the evolution of the age distribution which indicates that the set of constraints (3.7) are not violated, i.e. the number of employees is greater or equal to zero. It is noted that all adjustment values are discrete. However, lines are connected between points in the figure in order to facilitate the visualization. Table 3.1 reports the summation of the number of employees in each consecutive year. It is clearly seen that the 100-percent criteria are satisfied under the prescribed tolerance.

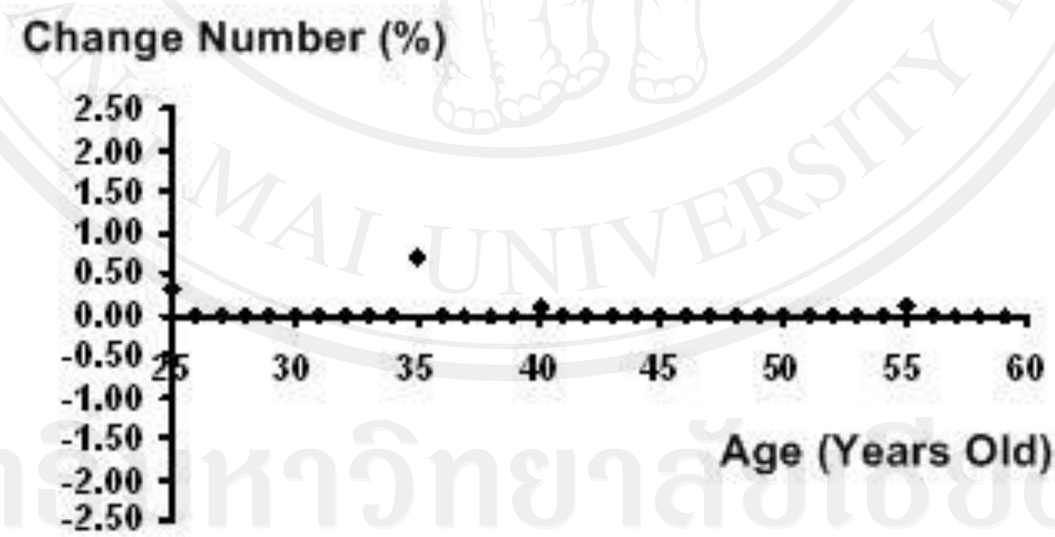



Figure 3.2. Adjustment magnitudes obtained from GA for study case 1.

A distinct discrepancy between the resulting age distribution and the desired one can be perceived in Figure 3.3. This suggests that the adjustment at only 4 specific age groups is not sufficient to alter the present age distribution to the desired one within the required time frame of 5 years. A larger number of age groups may be necessary for accelerating the adjustment process of the age distribution. Corresponding to this notion, the following adjustments are introduced: $\delta_{Aj}(t_i) = \delta_{Aj}$, where $j = 1,\ldots,35$. Thus, the adjustments are performed from the age of 25 years old to that of 59 years old. Consequently, total number of constraints *NCon* is equal to 185.

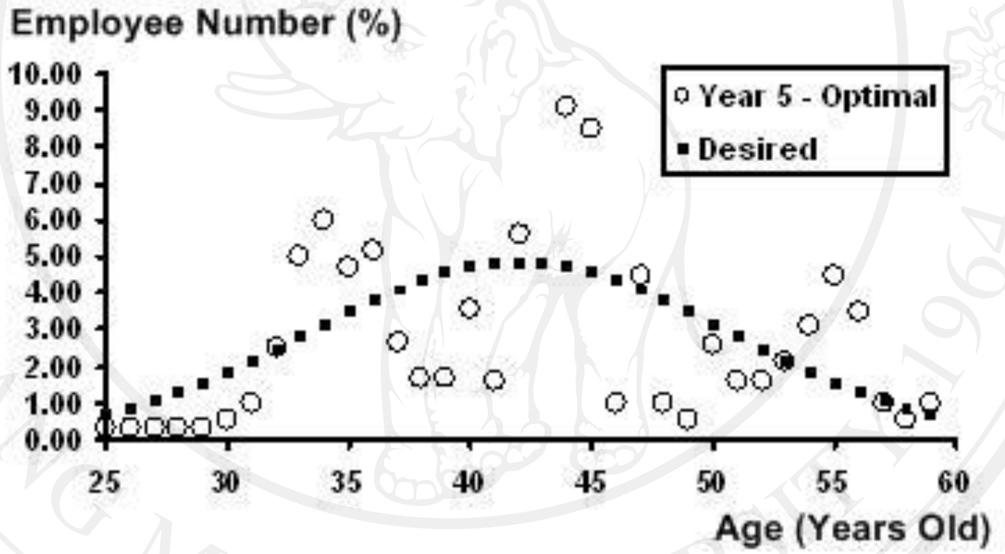



Figure 3.3. Age distribution at *NYear* = 5 for study case 1.

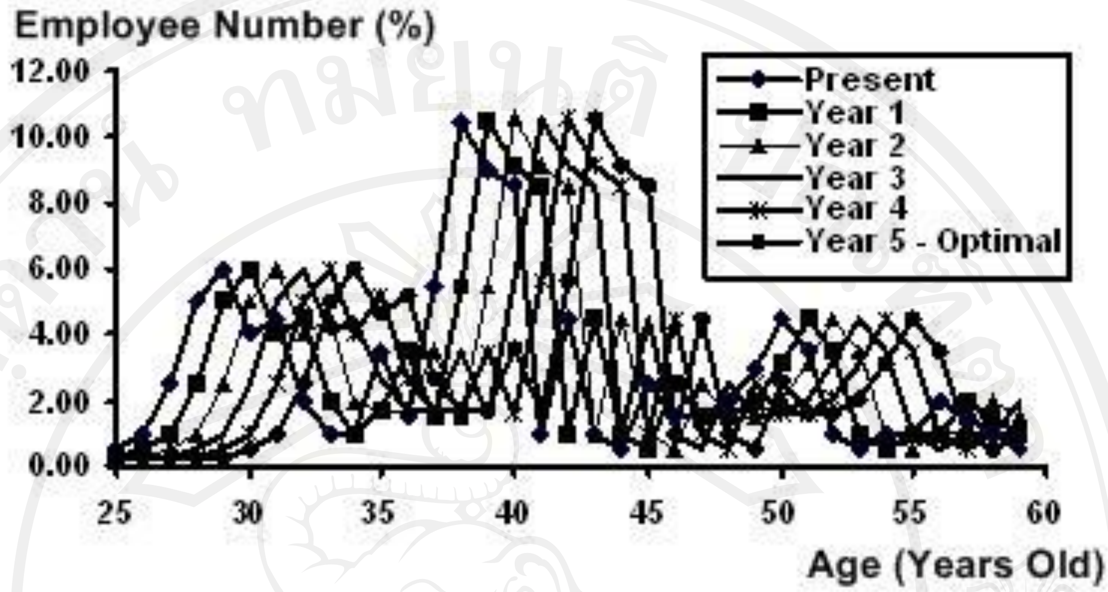



Figure 3.4. Evolution of age distribution for study case 1.

**Table 3.1 : Summation of the number of employees in each year for study case 1.**

| | $\sum_{j=1}^{35} E_{Aj}(t_i)$ |
|---|---|
| Present | 100.00 |
| Year 1 | 100.67 |
| Year 2 | 100.85 |
| Year 3 | 100.52 |
| Year 4 | 99.69 |
| Year 5 | 99.87 |

In this second case, the number of simulations of GA, the number of generations, and the population size are the same as in the first case. The best result from all simulations is taken as the adjustment magnitudes (see Figure 3.5) and is used to compute the age distribution at *NYear* = 5 (see Figure 3.6). All the results are shown in Figures 3.6 and 3.7, respectively. Figure 3.7 and Table 3.2 show that all the constraints are satisfied.

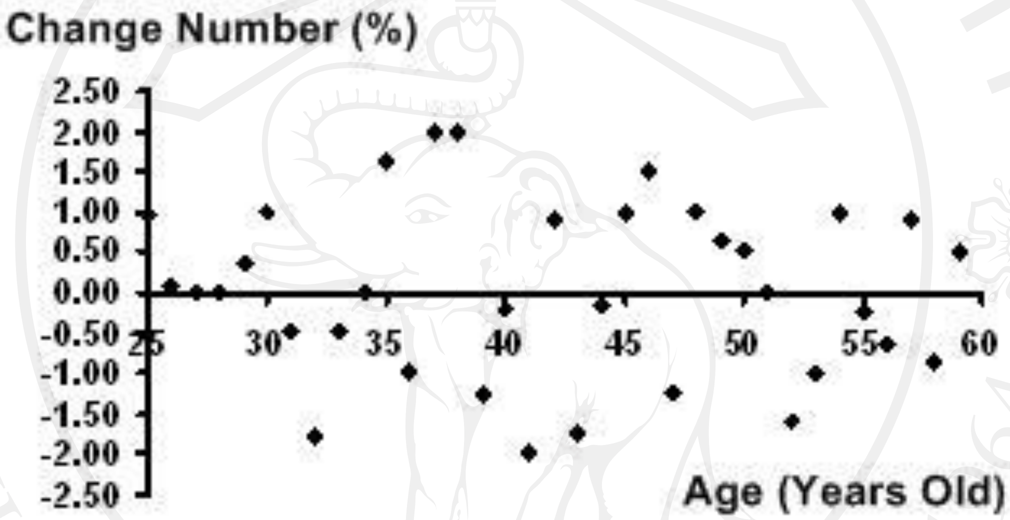



Figure 3.5. Adjustment magnitudes obtained from GA for study case 2.

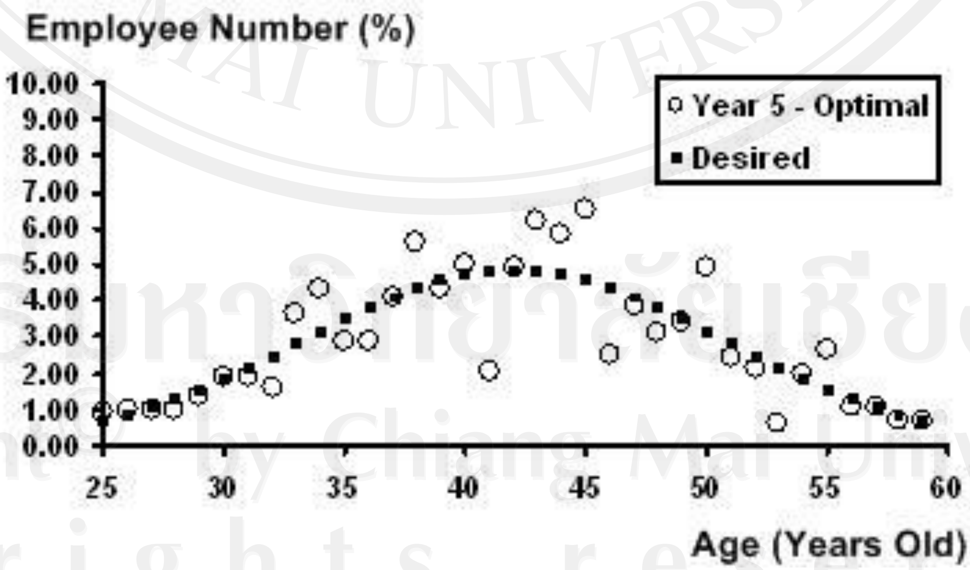



Figure 3.6. Age distribution at *NYear* = 5 for study case 2.

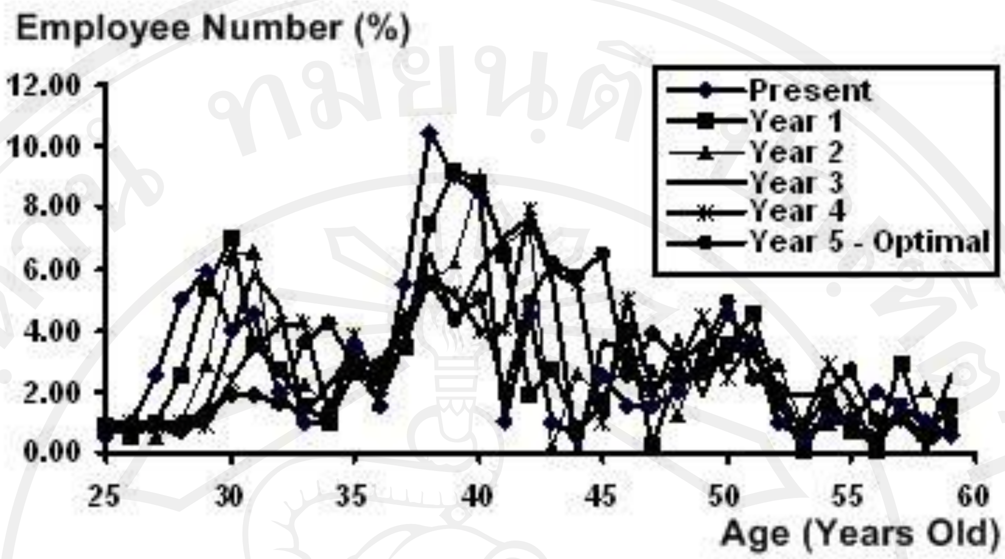



Figure 3.7. Evolution of age distribution for study case 2.

**Table 3.2 : Summation of the number of employees in each year for study case 2.**

| | $\sum_{j=1}^{35} E_{Aj}(t_i)$ |
|---|---|
| Present | 100.00 |
| Year 1 | 100.83 |
| Year 2 | 100.63 |
| Year 3 | 100.81 |
| Year 4 | 99.58 |
| Year 5 | 99.98 |

The comparison between both cases of study show that the adjustment process to the age distribution can be significantly improved when the adjustment of the number of employees is applied simultaneously on all age groups under a given time frame. However, the adjustment process according to the second case of study is rather hypothetical in that every age group is modified. Instead, a limited number of age groups should be considered. In the third case of study, the following adjustments are introduced: $\delta_{Aj}(t_i) = \delta_{Aj}$, where $j = 1,\ldots,31$. Thus, the adjustments are performed from the age of 25 years old to 55 years old. The corresponding total number of constraints *NCon* is equal to 165. GA is then employed to search for the optimal adjustment magnitudes. The number of simulations of GA, the number of generations, and the population size are the same as in the first case. Figures 3.8 to 3.10 show the numerical results for the third case of study.

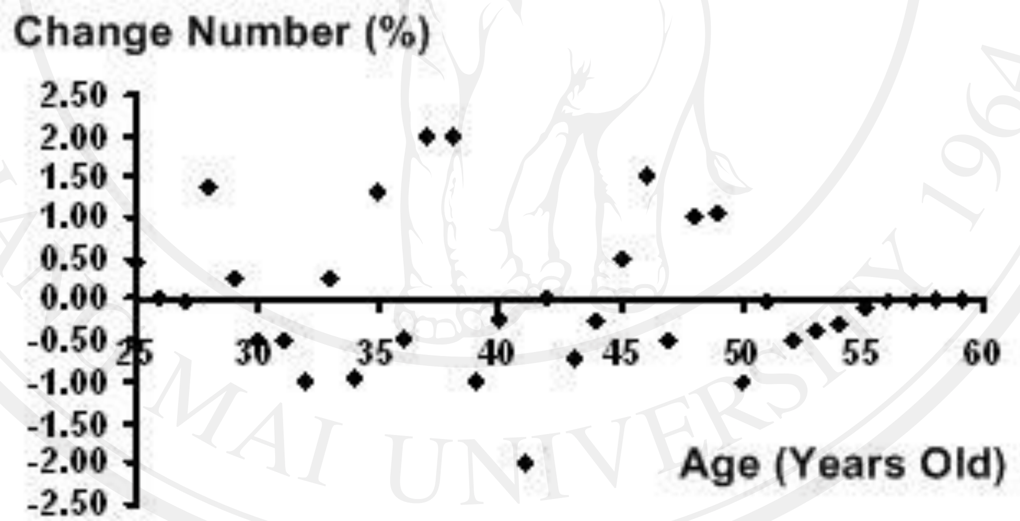



Figure 3.8. Adjustment magnitudes obtained from GA for study case 3.

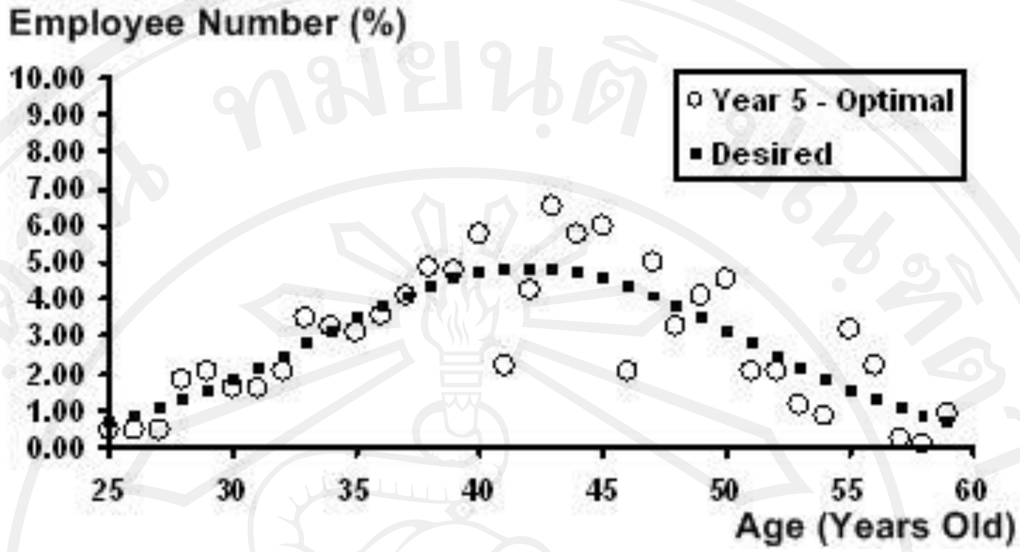



Figure 3.9. Age distribution at *NYear* = 5 for study case 3.

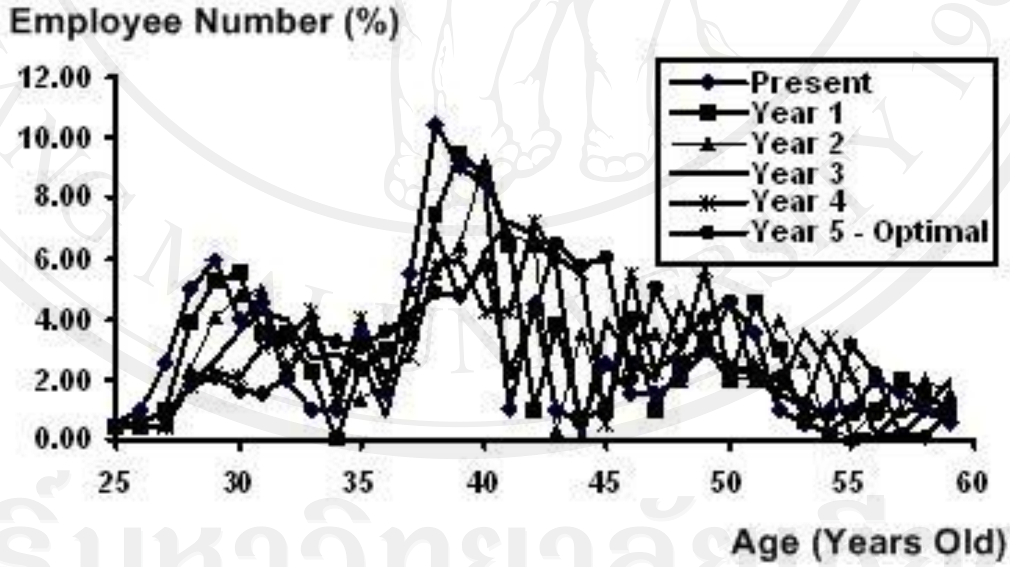



Figure 3.10. Evolution of age distribution for study case 3.

The performance of each respective HR planning scheme, i.e. the adjustment process, can be visualized using Figures 11 to 13, in which the histogram of the absolute deviation of the computed age distribution and the desire one are depicted, respectively. The histograms confirm that the second and third adjustment processes lead to better age distributions than that in the first case of study. There is no distinct difference in the absolute deviation magnitude between the second and the third case. In addition, the absolute deviation in those two cases lies in the lower order of magnitude when compared with the first case study. The absolute deviation magnitude in the two cases, i.e. second and third ones, is distributed mainly in the range of 0-1 % while the one in the first case is found the range of 0-4%. In this regard, the adjustment process with the adjustment magnitudes applied at a spectrum of age group is an alternative when the rapid convergence to the desired age distribution within a limited time frame is desired.

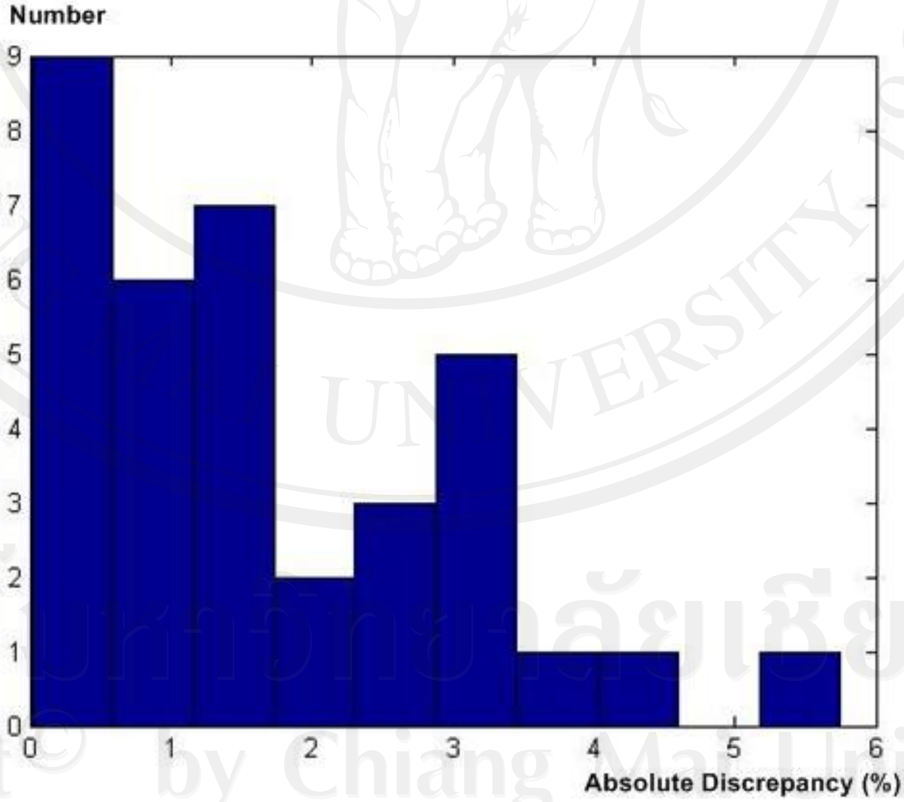



Figure 3.11. Histogram of absolute deviation for study case 1.

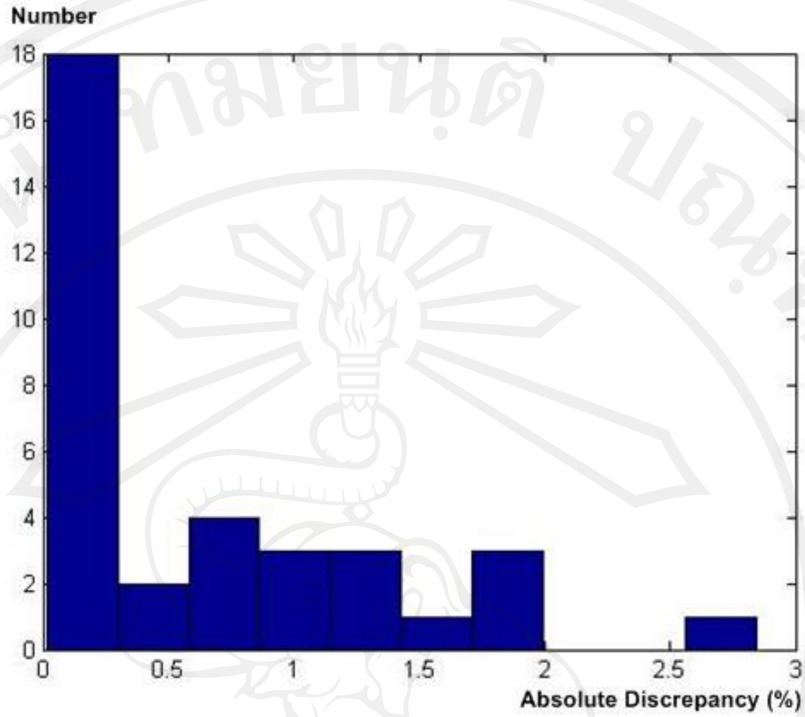



Figure 3.12. Histogram of absolute deviation for study case 2.

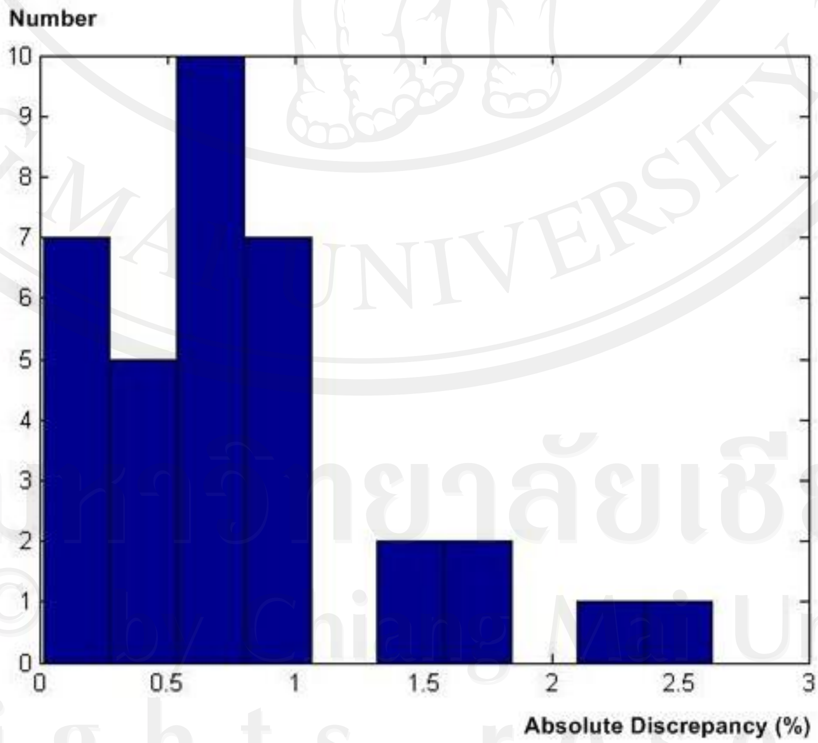



Figure 3.13. Histogram of absolute deviation for study case 3.

## 3.2 OPTIMAL LONG-TERM PLANNING OF KNOWLEDGE WORKERS

The long-term planning of knowledge workers is revisited again here. The optimization problem is formulated as follows:

Minimize $$O(\boldsymbol{S},\boldsymbol{D}) = \sum_{i-1}^{NYear}(SW_i - DW_i)^2 \qquad (i = 1,...,NYear) \qquad (3.12)$$

where,

***S*** : Vector of supplied knowledge workers in the *i*th year $SW_i$

***D*** : Vector of demanded knowledge workers in the *i*th year $DW_i$

$SW_i$ : Supplied knowledge workers in the *i*th year

$DW_i$ : Demanded knowledge workers in the *i*th year

*NYear* : Total number of years to be considered in the planning

, where $$\boldsymbol{S} = \begin{bmatrix} SW_1 & \cdots & SW_i & \cdots & SW_{NYear} \end{bmatrix}^T \qquad (3.13)$$

$$\boldsymbol{D} = \begin{bmatrix} DW_1 & \cdots & DW_i & \cdots & DW_{NYear} \end{bmatrix}^T \qquad (3.14)$$

Subject to: $$0 \le TC_i(\boldsymbol{S},\boldsymbol{D}) \le TBU_i \qquad (3.15)$$

$TBU_i$ : Upper bound of total budget available in the *i*th year

$TC_i$ : Total cost in the *i*th year

The evolution of the students at respective academic year in each year is:

$$ST2_i = \alpha_{1i} ST1_{i\text{-}1} \qquad (3.16)$$

$$ST3_i = \alpha_{2i} ST2_{i\text{-}1} \qquad (3.17)$$

$$ST4_i = \alpha_{3i} ST3_{i\text{-}1} \qquad (3.18)$$

$$G_i = \alpha_{4i} ST4_{i\text{-}1} \qquad (3.19)$$

$ST1_i$ : number of first academic-year students attending the universities in the *i*th year

$ST2_i$ : number of second academic-year students in the *i*th year

$ST3_i$ : number of third academic-year students in the *i*th year

$ST4_i$ : number of fourth academic-year students in the *i*th year

$G$ : number of graduates from the university in the *i*th year

$\alpha_{1i}$ : percentage of the first academic-year students that pass to the second year in the *i*th year

$\alpha_{2i}$ : percentage of the second academic-year students that pass to the third year in the *i*th year

$\alpha_{3i}$ : percentage of the third academic-year students that pass to the fourth year in the *i*th year

$\alpha_{4i}$ : percentage of the fourth academic-year students that graduate from the university in the *i*th year

The supply is described by

$$SW_i = SW_{i-1} + G_i - RW_i - RTW_i \tag{3.20}$$

In this optimization problem, the supplied knowledge workers in the *i*th year, i.e. $SW_i$ ($i$ = 1,...,$NYear$) are the design variables to be determined. The knowledge workers who resign and are retired from the work system in each year are given by

$$RW_i = \alpha_{RWi} SW_{i-1} \tag{3.21}$$

$$RTW_i = \alpha_{RTWi} SW_{i-1} \tag{3.22}$$

in which,

$\alpha_{RWi}$ : percentage of $RW_i$ with respect to $SW_{i-1}$

$\alpha_{RTWi}$ : percentage of $RTW_i$ with respect to $SW_{i-1}$

$RW_i$ : number of knowledge workers resigning from the work system in the *i*th year

$RTW_i$ : number of knowledge workers retiring from the work system in the *i*th year

The annual cost incurred by the education is

$$TC_i = FCOST_i * ST1_i + SCOST_i * ST2_i + TCOST_i * ST3_i + FTCOST_i * ST4_i \tag{3.23}$$

where

$FCOST_i$ : cost for educating a first year student in the *i*th year

$SCOST_i$ : cost for educating a second year student in the *i*th year

$TCOST_i$ : cost for educating a third year student in the *i*th year

*FTCOSTi* : cost for educating a fourth year student in the *i*th year

Finally, the initial conditions are

$$SW_0 = SW_o \tag{3.24}$$

$$ST1_0 = ST1_o \tag{3.25}$$

$$ST2_0 = ST2_o \tag{3.26}$$

$$ST3_0 = ST3_o \tag{3.27}$$

$$ST4_0 = ST4_o \tag{3.28}$$

in which

$SW_o$ : Supplied knowledge workers in the *starting* year

$ST1_o$ : number of first academic-year students in the *starting* year

$ST2_o$: number of second academic-year students in the *starting* year

$ST3_o$: number of third academic-year students in the *starting* year

$ST4_o$: number of fourth academic-year students in the *starting* year

It should be noted that the number of the first academic-year students attending the universities in respective years is the variable to be determine, i.e. $ST1_i$ ($i = 1,\ldots,NYear$).

According to the adaptive penalty GA used, the fitness function $F(\boldsymbol{S})$ is defined as

$$F(\boldsymbol{S}) = \begin{cases} 1/O(\boldsymbol{S},\boldsymbol{D}) & ;\ \boldsymbol{S} \text{ is feasible} \\ 1/\left[ O(\boldsymbol{S},\boldsymbol{D}) - \sum_{j=1}^{NYear} k_j v_j(\boldsymbol{S},\boldsymbol{D}) \right] & ;\ \boldsymbol{S} \text{ is infeasible} \end{cases} \tag{3.29}$$

The adaptive penalty scheme is given by

$$k_j = \left| max\left(O^{\inf}(\boldsymbol{S},\boldsymbol{D})\right) \right| \frac{< v_j(\boldsymbol{S},\boldsymbol{D}) >}{\sum_{l=1}^{NYear} [< v_l(\boldsymbol{S},\boldsymbol{D}) >]^2} \tag{3.30}$$

where $max(O^{inf}(\boldsymbol{S},\boldsymbol{D}))$ is the maximum of the objective function values in the current population in the infeasible region, $v_j(\boldsymbol{S},\boldsymbol{D})$ is the violation magnitude of the $j$th constraint. $\langle v_j(\boldsymbol{S},\boldsymbol{D})\rangle$ is the average of $v_j(\boldsymbol{S},\boldsymbol{D})$ over the current population. $k_j$ is the penalty parameter for the $j$th constraint defined at each generation. The violation magnitude is defined as

$$v_j(\boldsymbol{S},\boldsymbol{D}) = \begin{cases} \left|TC_j(\boldsymbol{S},\boldsymbol{D}) - TBU_j\right| & ; TC_j(\boldsymbol{S},\boldsymbol{D}) - TBU_j > 0 \\ 0 & \end{cases} \quad (3.31)$$

The following information is used in the numerical examples. The evolution of the $DW_i$ : demanded knowledge workers is given in Figure 3.14.

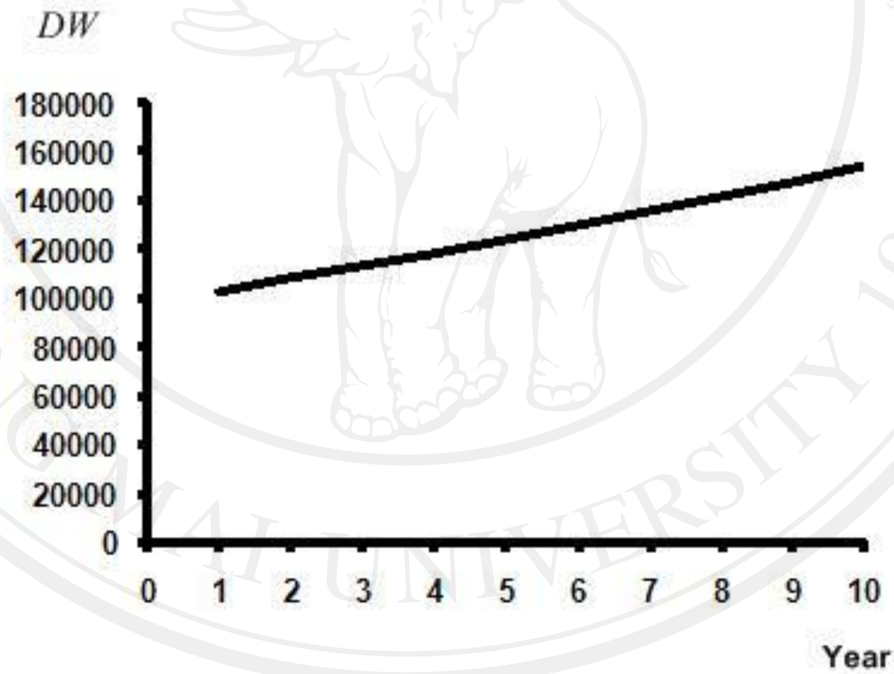



Figure 3.14. Demand of knowledge workers.

The development of the cost for each academic year is shown in Figure 3.15.

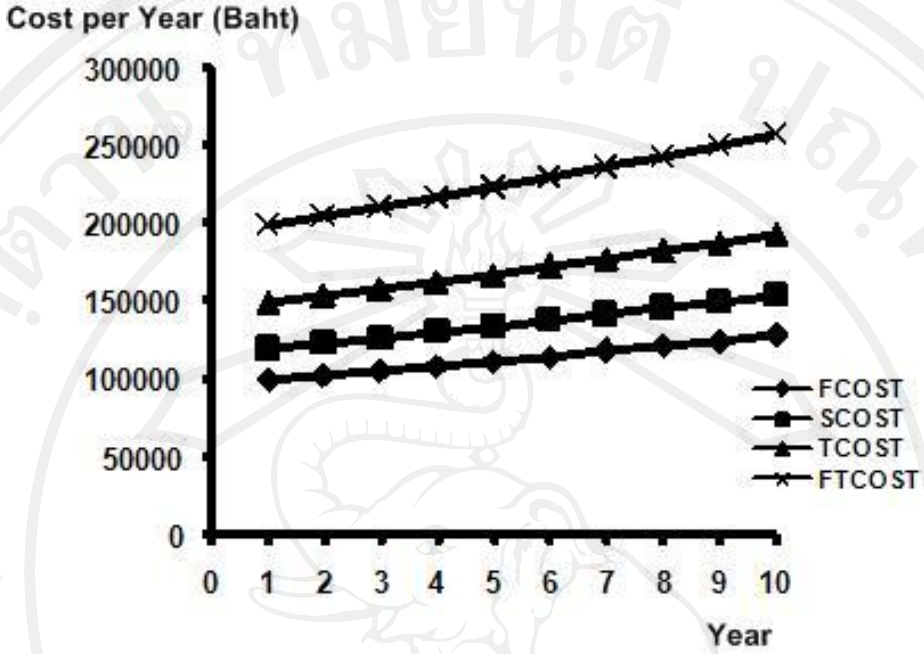



Figure 3.15. Costs for educating students at each respective academic-year.

The passing rate is shown in Figure 3.16.

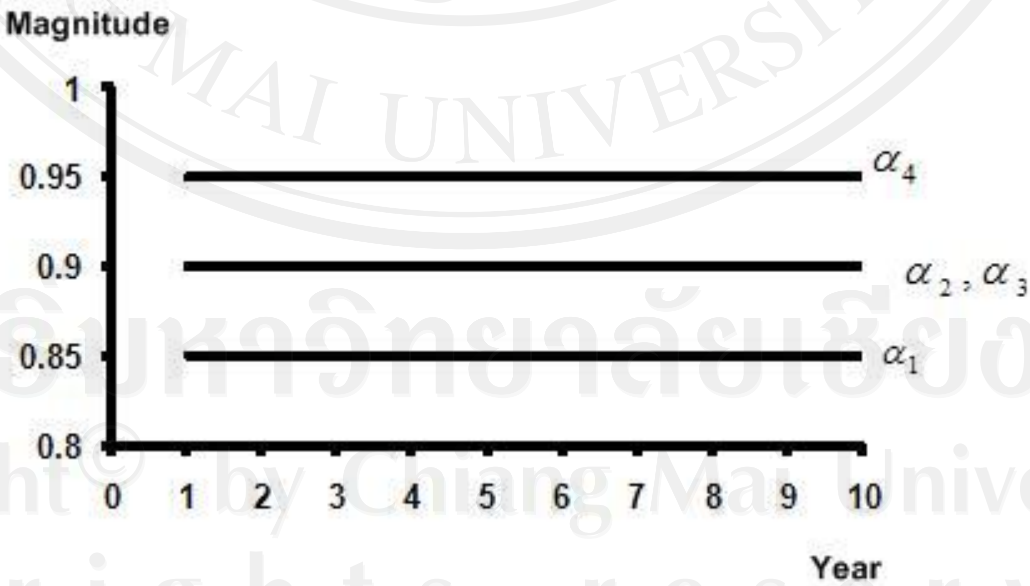



Figure 3.16. Passing rate.

Figure 3.17 and Figure 3.18 display the resigning rate and retiring rate, respectively. The upper bound of total budget available in each year is given in Figure 6. The initial conditions are as follow: $ST1_o$ = 3909, $ST2_o$ = 3788, $ST3_o$ = 3492, $ST4_o$ = 3173, and $SW_o$ = 10000. The total number of year is equal to 10, i.e. *NYear* = 10.

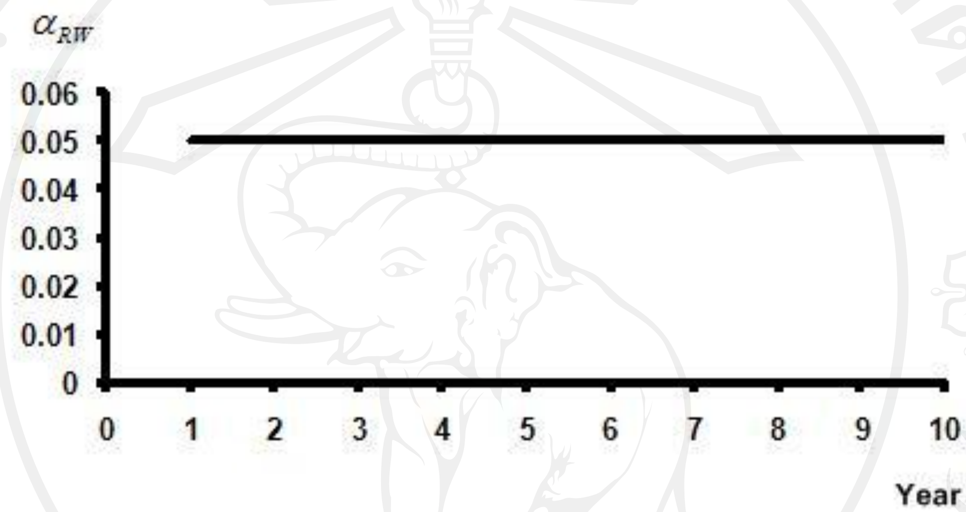



Figure 3.17. Resigning rate.

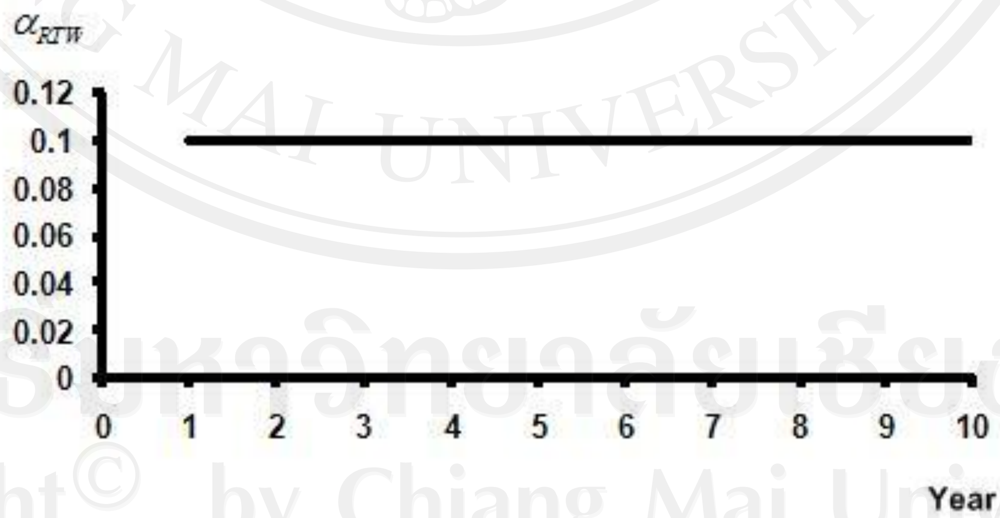



Figure 3.18. Retiring rate.

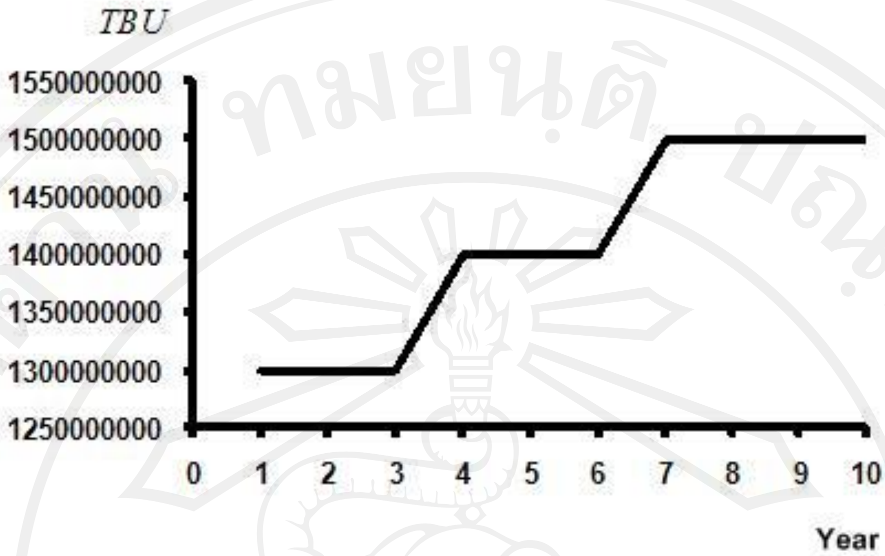



Figure 3.19. The upper bound of total budget available in each year.

GA is employed with the following parameters. The GA search uses a population size of 100. The number of generations used in the search is 100. A two-point crossover is utilized with a crossover rate of 0.8. The mutation rate is taken as 0.002. Figure 3.20 shows the solution of the optimization problem.

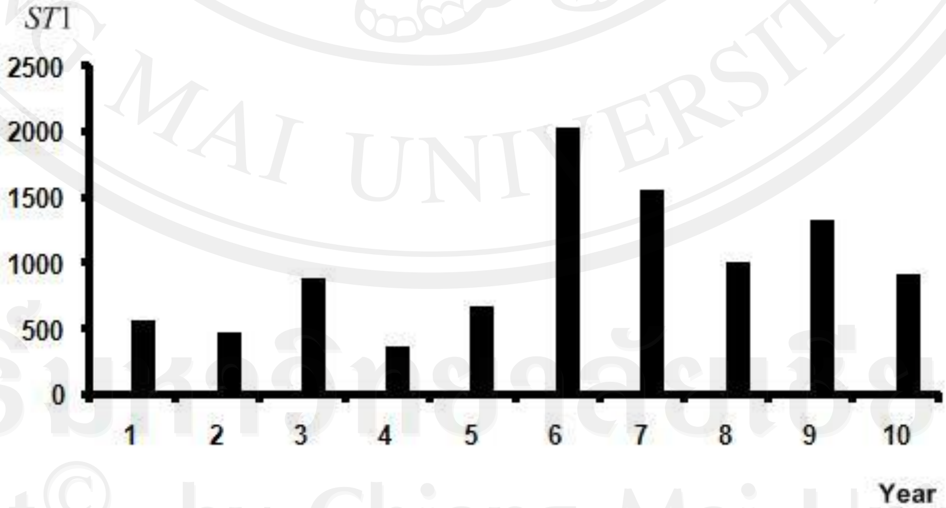



Figure 3.20. The optimal number of the first academic-year students that can be supported.

## 3.3 OPTIMAL DESIGN OF APPOINTMENT SYSTEM FOR OUTPATIENT SERVICE

The optimal design of doctor appointment system is considered here again. The arrival time of each consultation case is classified into two types. For the first consultation case, the arrival time is related to the appointment time as follows:

$$A_{ij} = t_i + \Delta_{ij} \quad (3.32)$$

where $\Delta_{ij}$ is the time deviating from the appointment time $t_i$. The deviation time can be random and thus treated as a random variable. The punctuality of an appointed patient is interpreted from the condition

$$\Delta_{ij} \begin{cases} < 0 & \text{early arrival} \\ = 0 & \text{punctual arrival} \\ > 0 & \text{late arrival} \end{cases} \quad (3.33.1)$$

When considering that the earliness or waiting prior to appointment time is not a consequence of the appointment system as in [Cayirli and Veral 2003], then $\Delta_{ij}$ is defined as

$$\Delta_{ij} \begin{cases} = 0 & \text{early and punctual arrival} \\ > 0 & \text{late arrival} \end{cases} \quad (3.33.2)$$

For the second consultation case, the arrival time is given by

$$A_{ij} = EF_{ij} + TL_{ij} \quad (3.34)$$

where $EF_{ij}$ is the ending service time after the first consultation and $TL_{ij}$ is the time required for the laboratory tests of that patient, respectively. The ending service time after the first consultation can be computed from Eq. (3.36).

The starting service time $B_{ij}$ is obtained from

$$B_{ij} = max\left(A_{ij}, E_{(i-1)j}\right) \qquad ;\ i = 2,\ldots, NP_j \tag{3.35.1}$$

and

$$B_{1j} = max\left(A_{1j}, t_1\right) \tag{3.35.2}$$

which reflects the fact that the first patient to each doctor can have the healthcare service only after the starting office hour.

The ending service time of each consultation case, i.e. first or second consultation, is defined as

$$E_{ij} = B_{ij} + L_{ij} \tag{3.36}$$

$L_{ij}$ is equal to zero if the *i*th-block patient under the *j*th doctor is absent or no-show. In addition, when the patient under consideration requires the second consultation and $E_{ij}$ corresponds to the ending service time after the first consultation, then $E_{ij}$ is further used as $EF_{ij}$ for the computation of the arrival time for the corresponding second consultation case. That is

$$EF_{ij} = E_{ij} \tag{3.37}$$

for its used in Eq. (3.33). It should be noted that the length of service time $L_{ij}$ is separated into two cases in all mathematical expressions. In the first consultation case, the length of service time for the first consultation $L1_{ij}$ must be used for $L_{ij}$, i.e. setting

$$L_{ij} = L1_{ij} \tag{3.38.1}$$

The second consultation case fixes the length of service time for the second consultation $L2_{ij}$ for $L_{ij}$

$$L_{ij} = L2_{ij} \tag{3.38.2}$$

Next the relevant performance indices will be defined. First, the waiting time $W_{ij}$ of the $i$th-block of consultation case (either first or second) under the $j$th doctor is

$$W_{ij} = max(0, B_{ij} - A_{ij}) \tag{3.39}$$

The total waiting time corresponding to the service from the $j$th doctor $W_j$ is

$$W_j = \sum_{i=1}^{NP_j} W_{ij} \tag{3.40}$$

The total waiting time in the appointment system $W_T$ is thus

$$W_T = \sum_{j=1}^{nD} W_j \tag{3.41}$$

The average waiting time of a patient $W_A$ is

$$W_A = \frac{1}{N_P n_D} W_T \tag{3.42}$$

where

$$N_P = \sum_{j=1}^{nD} NP_j \tag{3.43}$$

The overtime of the $j$th doctor $OT_j$ is obtained from

$$OT_j = max(0, E_{NP_j j} - t_f) \tag{3.44}$$

where $E_{NP,j}$ is the ending service time of the last consultation case under the $j$th doctor. The definition of the $j$th-doctor overtime implies that there is no overtime if the doctor finishes the work before the office hour.

The total overtime in the appointment system $OT_T$ is

$$OT_T = \sum_{j=1}^{nD} OT_j \tag{3.45}$$

The average overtime for a doctor $OT_A$ is

$$OT_A = \frac{1}{n_D} OT_T \tag{3.46}$$

The $j$th doctor idle time incurred just before the arrival of the $i$th-block of consultation case (either first or second) is

$$IT_{ij} = max(0, A_{ij} - E_{(i-1)j}) \qquad ; \; i = 2, \ldots, NP_j \tag{3.47.1}$$

and

$$IT_{1j} = max(0, A_{1j} - t_1) \tag{3.47.2}$$

The total idle time of the $j$th doctor $IT_j$ is

$$IT_j = \begin{cases} \sum_{i=1}^{NP_j} IT_{ij} & ; OT_j \geq 0 \\ \sum_{i=1}^{NP_j} IT_{ij} + |OT_j| & ; OT_j < 0 \end{cases} \tag{3.48}$$

The inclusion of the overtime term into the computation of the total idle time suggests that the free time of the doctor before the end of the office hour be considered as an idle time as well.

The total idle time in the appointment system $IT_T$ is

$$IT_T = \sum_{j=1}^{nD} IT_j \tag{3.49}$$

The average idle time for a doctor $IT_A$ is

$$IT_A = \frac{1}{n_D} IT_T \tag{3.50}$$

It should be noted that the performance indices as defined above can be combined in a various ways to establish the performance functions of the appointment system. As an example, the performance of an appointment system is measured through the expected total cost of appointment system $E[C_T]$ as defined by

$$E[C_T] = c_W E[W_T] + c_{OT} E[OT_T] + c_{IT} E[IT_T] \tag{3.51}$$

where $c_W$, $c_{OT}$, and $c_{IT}$ is the cost per time unit associated to $W_T$, $OT_T$, and $IT_T$, respectively. The symbol $E[f(\boldsymbol{X})]$ denotes the expectation of a function $f(\boldsymbol{X})$. A comprehensive collection of performance measures used in the literature can be found in [Cayirli and Veral 2003].

The optimal design of this appointment system is defined as follows:

$$\min_{(n_D, \Delta t_{\text{block}})} E[C_T(n_D, \Delta t_{\text{block}})]_{Npatient=50} = c_W E[W_T(n_D, \Delta t_{\text{block}})]_{Npatient=50} + c_{OT} E[OT_T(n_D, \Delta t_{\text{block}})]_{Npatient=50} + c_{IT} E[IT_T(n_D, \Delta t_{\text{block}})]_{Npatient=50} \tag{3.52}$$

Subject to

$$1 \le n_D \le 50 \tag{3.53}$$

$$\Delta t_{\text{block}} > 0 \quad (3.54)$$
$$E\left[W_A\left(n_D, \Delta t_{\text{block}}\right)\right]_{Npatient=50} \leq \delta_W \quad (3.55)$$
$$E\left[OT_A\left(n_D, \Delta t_{\text{block}}\right)\right]_{Npatient=50} \leq \delta_{OT} \quad (3.56)$$
$$E\left[IT_A\left(n_D, \Delta t_{\text{block}}\right)\right]_{Npatient=50} \leq \delta_{IT} \quad (3.57)$$

where $\delta_W$, $\delta_{OT}$, and $\delta_{IT}$ are the thresholds of the average waiting time of a patient, the average overtime for a doctor, and the average idle time for a doctor, respectively. It should be noted that all expectations are conditional on a fixed number of patients $N_{patient}$ equal to 50.

The objective of the design is to determine the number of doctors $n_D$ and the appointment interval $\Delta t_{\text{block}}$ such that the expected total cost of appointment system $E[C_T]$ is minimized under the constraints. The design variables that minimize the objective function and at the same time satisfy the constraints will be referred to as the optimal number of doctors $n_D^*$ and the optimal appointment interval $\Delta t_{\text{block}}^*$. The expectation of total cost and other variables like $W_T$, $OT_T$, and $IT_T$ signify that these are random variables. This is because $W_T$, $OT_T$, and $IT_T$ are the functions of the random variables shown in Table 3.3. $c_W$, $c_{OT}$, and $c_{IT}$ are set equal to 100, 600, and, 300, respectively.

The appointment system of this exemplified outpatient department assumes that the total number of outpatients receiving the service is equal to 50. The absence or no-show probability of each patient $p_{abs}$ is equal to 0.20. The probability that an appointed patient will have laboratory tests $p_{lab}$ is equal to 0.40. The ending time of the office hour $t_f$ is equal to 180. The length of service time for the 1st-consultation $L1_{ij}$, the length of service time for the 2nd-consultation $L2_{ij}$, the time required for the laboratory tests $TL_{ij}$, and the time deviating from the appointment time $\Delta_{ij}$ are treated as independent random variables whose distribution and associated parameters are given in Table 1.

**Table 3.3 : Definition of random variables in the numerical example.**

| Random Variable | Distribution (minutes) |
|---|---|
| Length of Service Time for The 1st-consultation ($L1_{ij}$) | Uniform(10,20) |
| Length of Service Time for The 2nd-consultation ($L2_{ij}$) | Uniform(7,12) |
| Time Required For The Laboratory Tests ($TL_{ij}$) | Triangular(10,20,30) |
| Time Deviating From The Appointment Time ($\Delta_{ij}$) | Uniform(0,10) |

GAs is used for determining $n_D^*$ and $\Delta t_{\text{block}}^*$. Since it is the minimization problem, the fitness function (23) is defined as

$$F(n_D,\Delta t_{\text{block}})=\begin{cases} 1\big/E[C_T(n_D,\Delta t_{\text{block}})]_{Npatient=50} & ;(n_D,\Delta t_{\text{block}})\text{ is feasible} \\ 1\Big/\left(E[C_T(n_D,\Delta t_{\text{block}})]_{Npatient=50}+\sum_{j=1}^{3}k_j v_j(n_D,\Delta t_{\text{block}})\big|_{Npatient=50}\right) & ;(n_D,\Delta t_{\text{block}})\text{ is infeasible}\end{cases} \quad (3.58)$$

in which $E[C_T(n_D,\Delta t_{\text{block}})]$ follows the objective function (27). The penalty functions, according to the constraints (30) to (32), are respectively

$$v_1(n_D,\Delta t_{\text{block}})\big|_{Npatient=50}=\begin{cases}\left|E[W_A(n_D,\Delta t_{\text{block}})]_{Npatient=50}-\delta_W\right| & ;E[W_A(n_D,\Delta t_{\text{block}})]_{Npatient=50}-\delta_W>0 \\ 0 & ;\text{otherwise}\end{cases} \quad (3.59)$$

$$v_2(n_D,\Delta t_{\text{block}})\big|_{Npatient=50}=\begin{cases}\left|E[OT_A(n_D,\Delta t_{\text{block}})]_{Npatient=50}-\delta_{OT}\right| & ;E[OT_A(n_D,\Delta t_{\text{block}})]_{Npatient=50}-\delta_{OT}>0 \\ 0 & ;\text{otherwise}\end{cases} \quad (3.60)$$

$$v_3\left(n_D, \Delta t_{\text{block}}\right)\Big|_{Npatient=50} = \begin{cases} \left| E\left[IT_A\left(n_D, \Delta t_{\text{block}}\right)\right]_{Npatient=50} - \delta_{IT} \right| & ; E\left[IT_A\left(n_D, \Delta t_{\text{block}}\right)\right]_{Npatient=50} - \delta_{IT} > 0 \\ 0 & ; \text{otherwise} \end{cases}$$

(3.61)

The values of the thresholds are given in Table 3.4.

**Table 3.4 : Threshold values in case of deterministic number of patients.**

| Parameter | Magnitude (minutes) |
|---|---|
| $\delta_W$ | 5 |
| $\delta_{OT}$ | 30 |
| $\delta_{IT}$ | 30 |

It should be noted that the fitness function involves the expected values. The relevant expected values appear in the objective function and the constraints. This means that there must be the computation of the expected values for each individual of the chromosome population considered in the successive generations of the search. Monte Carlo Simulation (MCS) is run for each individual chromosome to obtain its corresponding value of the fitness function. A sample of all random variables is generated according to their probabilistic descriptions and the evaluation of the required expected values is carried out for each chromosome. The execution of MCS for each individual of the chromosome population would be very time consuming and inefficient for a considerable sample size. A more efficient solution to this problem follows from the consideration that the potential chromosomes appear a large number of times in the successive generations during the GA search whereas the less potential solutions are gradually eliminated. Only a limited sample size is therefore necessary for MCS. The MCS sample size for the evaluation of the expected values in this example is thus set equal to 20.

GAs search employs the population size of 50. The number of generations used in the search is 50. A two-point crossover is utilized with the crossover rate of 0.80. The mutation rate is taken as 0.002.

Figure 3.21 shows the search history. The optimal number of doctors $n_D^*$ is equal to 5. The optimal appointment interval $\Delta t_{\text{block}}^*$ is equal to 14 minutes. The distributions of chromosomes at various generations are shown in Figure 3.22.

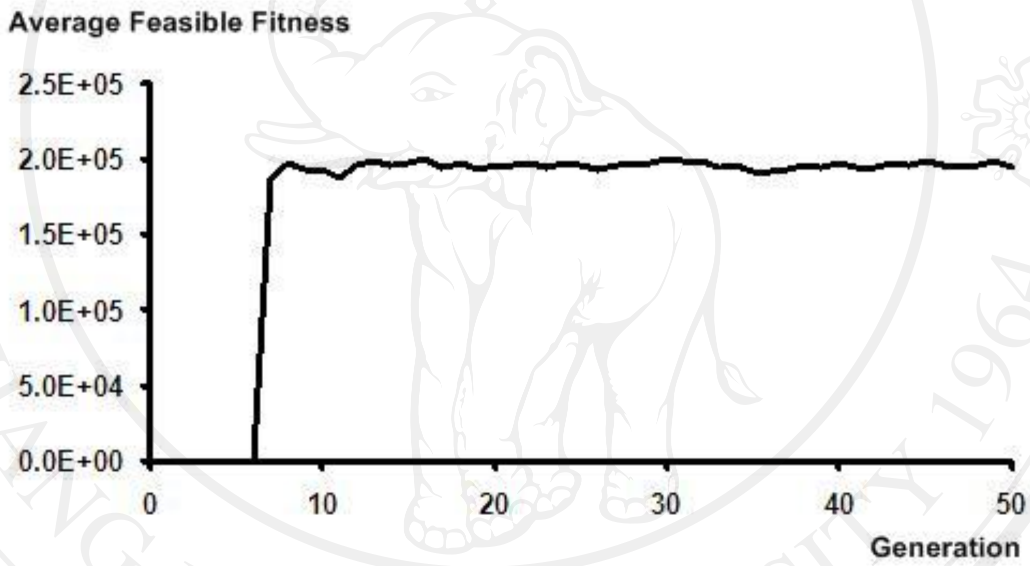
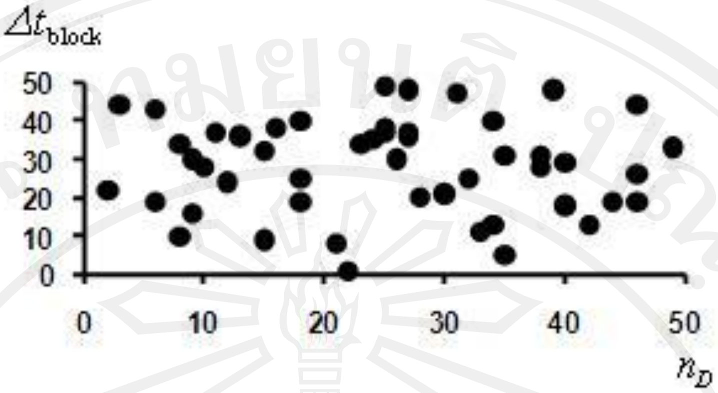
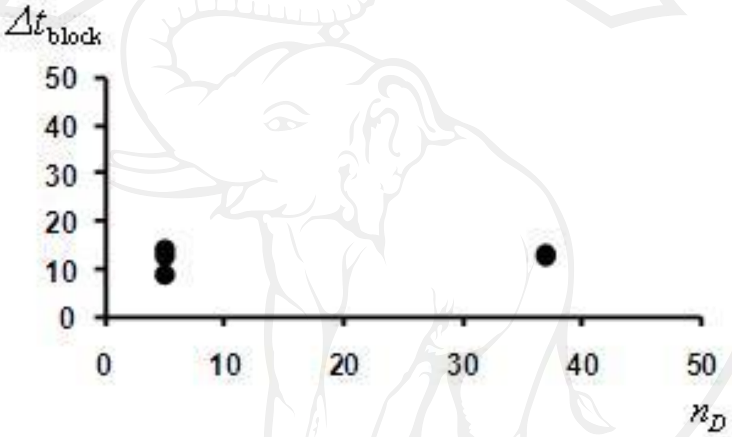


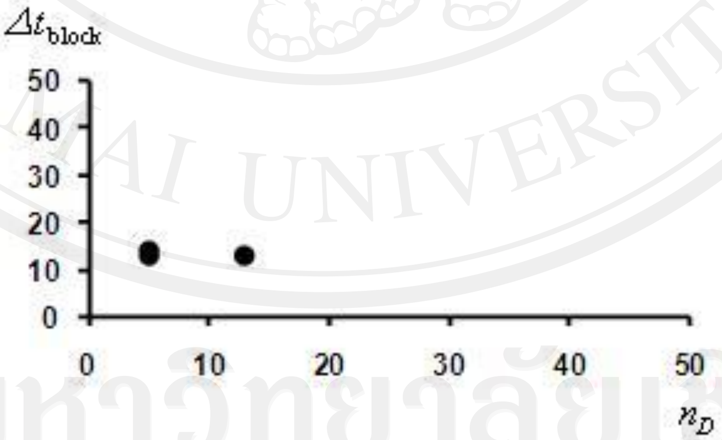

Figure 3.21. The history of the average fitness of the feasible chromosomes in case of deterministic number of patients.




(a)




(b)




(c)

Figure 3.22. The distributions of chromosomes at various generations:
(a) – Starting Generation, (b) – 20*th* Generation, and (c) – 50*th* Generation.

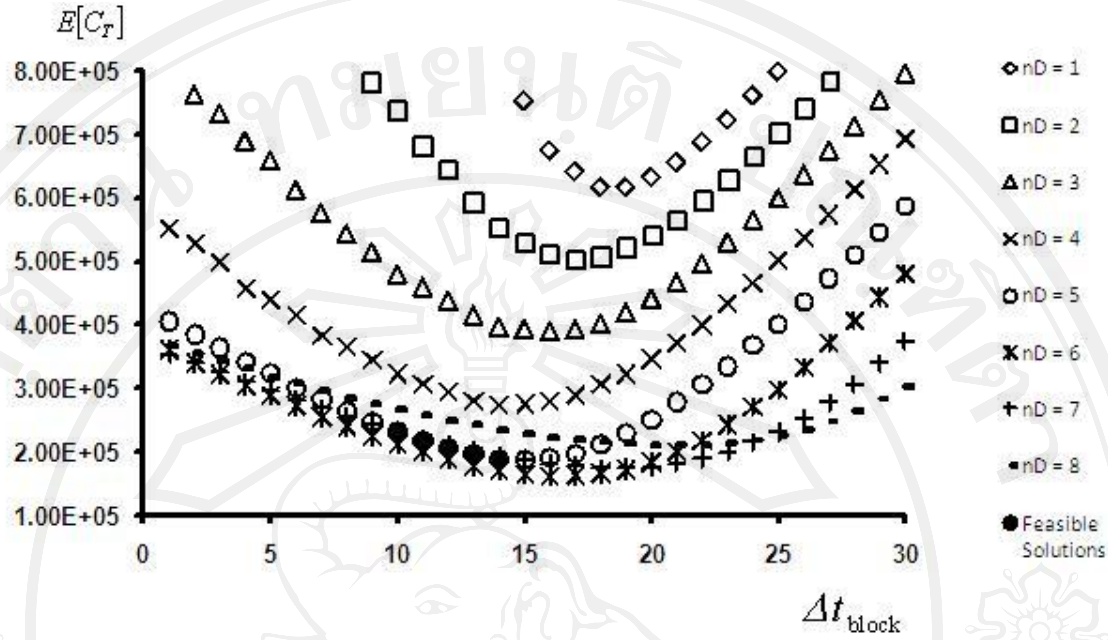



Figure 3.23. Verification of the optimal design from GA search in case of deterministic number of patients.

The results obtained from GAs have been verified with the simulation. The verification is shown in Figure 3.23. It should be noted that there are only 5 feasible solutions among all possible combinations. All 5 feasible solutions belong to the case of 5 doctors. The solutions are classified as feasible when they satisfy all the constraints. It should be noted that the constraints in this numerical example are complicated and can be obtained in terms of numerical values only. Nevertheless, the optimization result shows that the employed method of constraint handling effectively performs to obtain the optimal solution.

## 3.4 DISCUSSIONS

Although complicate mathematical models are formulated herein, the following technical aspects should be noted. First, it is shown here that GA reveals satisfactory performance in context of dynamic systems or time-variant problems where the objective function in general can be an implicit function of the variables to be optimized. Second, the total number of variables to be optimized can be higher than that considered in the numerical examples, when non-constant adjustment magnitudes are considered for respective age groups. Third, the utilized adaptive penalty scheme works satisfactorily for sufficiently large numbers of constraints. Interestingly, the number of constraints is relatively high when compared with many other optimization problems. Such a large number of constraints are attributed by the dynamic aspect of the problem, from which the constraints are imposed at every time step. Yet, there are many constraints at each time step. Total number of constraints is even dramatically increased when the number of time steps becomes high. Since the constraints that are considered herein are limited to those relevant to the age only, it is expectable that the number of constraints becomes extremely high in practical HRM. Other kinds of constraint include the financial constraints, performance constraints, merit constraints, etc. Therefore, constraint handling is a critical issue in the application of GA to HRM. Third, the adjustment magnitudes that are used in the determination of the age distribution are selected from the best GA solution. There are other GA solutions that yield the same order of total discrepancy magnitude (ERR). In other words, there are other alternative sets of adjustment magnitudes. The number of alternative sets can be filtered out down to a smaller number of sets by imposing additional constraints. With respect to other possible alternative sets, HR planning for HRM is a multi-modal optimization. Therefore, GA for multi-modal optimization is required when several alternative sets of the adjustment magnitudes are desired. Such a case is out of the scope of this research and thus will not be addressed further.