CHAPTER 3

RELEVANT THEORY

3.1 Theory

3.1.1 Capital Asset Pricing Model: CAPM

The CAPM builds on the model of portfolio choice was developed by Markowitz (1959). In Markowitz's model, investors select a portfolio at time *t-1* that produces a stochastic return at *t*. The model assumes that investors are risk averse. When they choose portfolios, they care only about the mean and variance of their one-period investment return. As a result, investors choose "mean-variance-efficient" portfolios, in the sense that the portfolios:

1) Minimize the variance of portfolio return, given expected return

2) Maximize expected return, given variance.

Thus, the Markowitz approach is generally called a "mean-variance model" or an "efficient E-V frontier."

The capital asset pricing model (CAPM) of Sharpe (1964) and Lintner (1965) marks the birth of asset pricing theory (resulting in a Nobel Prize for Sharpe in 1990). Four decades later, the CAPM is still widely used in applications, such as estimating the cost of capital for firms and evaluating the performance of managed portfolios. The attraction of the CAPM involves with its powerful and intuitively pleasing predictions about how to measure the risk and the relation between expected return and risk. (Fama & French, 1968)

The portfolio model provides an algebraic condition on asset weights in mean-variance efficient portfolios. The CAPM turns this algebraic statement into a testable prediction about the relation between risk and expected return by identifying a portfolio that must be efficient if asset prices are clarified.

For the **CAP-model**[6], it is a ceteris paribus model which is only valid within a special set of assumptions. The assumptions of the CAPM are:

1. Investors are risk averse individuals who maximize the expected utility of their end of period wealth. Implication: The model is a one period model.
2. Investors have homogenous expectations (believe) about asset returns. Implication: all investors perceive identical opportunity sets. In the other words, everyone has the same information at the same time.
3. There exists a risk free asset and investors that may borrow or lend unlimited amounts of this asset at a constant rate: the risk free rate (k_f).
4. There are definite numbers of assets and their quantities are fixed within the one period world.
5. All assets are perfectly divisible and priced in a perfectly competitive market. Implication: e.g. human capital is non-existing (it is not divisible and it can't be owed as an asset).
6. Asset markets are frictionless and the information is costless and simultaneously available to all investors. Implication: the borrowing rate equals the lending rate.

[6] See Capital Asset Pricing Model. (2008) for a detailed definition.

7. There are no market imperfections such as taxes, regulations, or restrictions on short selling.

The CAPM is a model for pricing an individual security or a portfolio. For individual securities, we made use of the security market line (SML) and its relationship to expect return *E(R)* and systemic risk (beta (β)) to show how the market must price each security in relation to their security risk class.

The SML enables us to calculate the reward-to-risk ratio for each security in the market. Therefore, when the expected rate of return for any security is deflated by equation's beta (β) coefficient, the reward-to-risk ratio for any individual securities in the market is equal to the market reward-to-risk ratio. Thus:

$$\frac{E(R_i)-R_f}{\beta_i} = E(R_m) - R_f \qquad (1)$$

The market reward-to-risk ratio is effective to the market risk premium and by rearranging the above equation and solving for $E(R_i)$; we obtain the Capital Asset Pricing Model (CAPM).

$$E(R_i) = R_f + (E(R_m) - R_f)\beta_i \qquad (2)$$

By: $E(R_i)$ is the expected return on the capital asset

R_f is the risk-free rate of interest for example interest arising from government bonds.

β_i is the sensitivity of the asset returns to market returns, or also

$$\beta_i = \frac{Cov(R_i, R_m)}{Var\,(R_m)} \tag{3}$$

$E(R_m)$ is the expected return of the market

$E(R_m) - R_f$ is sometimes known as the market premium or risk premium (the difference between the expected market rate of return and the risk-free rate of return)

Restated, in terms of risk premium, we found that:

$$E(R_i) - R_f = \beta_i(E(R_m) - R_f) \tag{4}$$

, which states the individual risk premium equals to the market premium time's beta.

3.1.2 Security Market Line (SML)[7]

The Security Market Line (SML) is the graphical representation of the Capital Asset Pricing Model. It displays the expected rate of return for overall market as a function of systematic (non-diversifiable) risk (beta (β)).

SML (also known as a characteristic line) is the graphical representation of CAPM. Knowing more about CAPM, SML is a straight sloppy line which gives the relationship between expected rate of return and market risk (or systematic risk) of

[7] See Securities Market Line (2008) from http://www.nobletrading.com/blogs/2008/09/what-is-security-market- line- or-sml.html for a review of SML.

overall market. The Y-Intercept (beta (β) =0) of the SML is equal to the risk-free interest rate. The slope of the SML is equal to the Market Risk Premium and reflects investors' degree of risk aversion at a given time.

The X-axis of the security market line represents the market risk or beta (β) and the Y-axis of SML represents the expected market return $E(R_m)$ in percentage at a point of time. The rate of risk free investments is represented as a line parallel to X-axis and the SML starts from there.

When it is used in portfolio management, a single asset is plotted against the SML using its own beta and historical rate of return. If the plot of the asset falls above the SML, it is considered to be a good rate of return relates to its risk, and vice versa if it falls below.(Durlauf & Blume, 2008)

Then, $$E(R_i) = R_f + (E(R_m) - R_f)\beta_i \quad (2)$$

Expected Return on the Market

$$\bar{R}_M = R_F + Market\ Risk\ Premium \quad (5)$$

Expected return on an individual security

$$\bar{R}_i = R_F + \beta_i \times \underbrace{(\bar{R}_M - R_F)}_{\text{Market Risk Premium}} \quad (6)$$

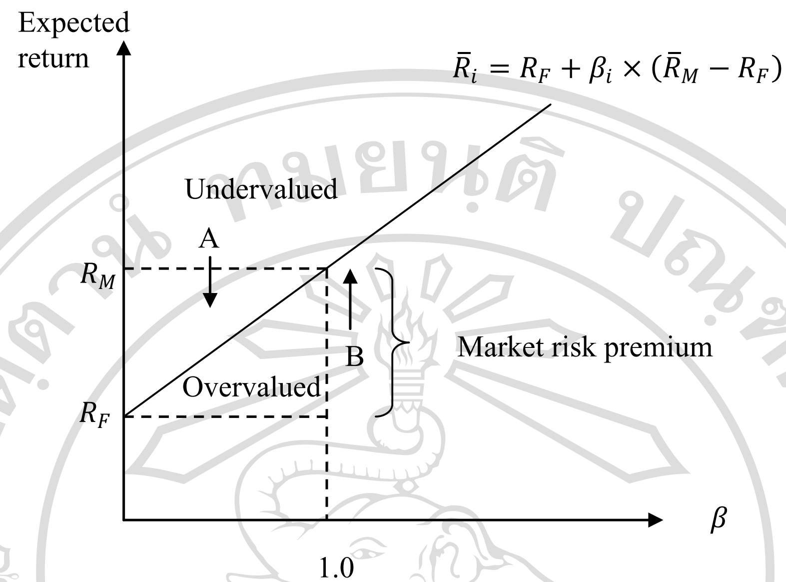

Figure 3.1: Security Market Line

Source: Fama, E., & French, K. (1968).

For example, if the risk-free ratio is 4%, the beta value of market is 3% and expected return from market $E(R_m)$ is 10%, then expected return will be 4+3(10-4) = 22% **(equation (2))**[8]; and SML will start from 4% at Y-axis and it will pass through 22% when beta is 3.

Security market line is a simple but powerful tool for finding return and risk associated with a portfolio. Investors can plot individual stock's beta and expected return against SML. If the expected return from the stock is above SML, the stock is considered to be undervalued and it is predicted to offer good return for the risk taken. If the expected return falls below SML, the stock is considered to be overvalued and it is predicted to offer less return for the risk taken.(Phillips, 2008)

[8] $E(R_i) = R_f + \big(E(R_m) - R_f\big)\beta_i$, From equation (2)

From equation (2), we can consider the beta coefficient as follow:

If beta equal one, stock is average risk. If market rate of return change 100% investor will get expected rate of return from asset group index change at 100%.

If beta greater than one, stock is riskier than average. If market rate of return change 100% investor will get expected rate of return from asset group index change greater than 100%.

If beta less than one, stock is less risky than average. If market rate of return change 100% investor will get expected rate of return from asset group index change lower than 100%.

From figure 3.1, stock A locates over SML line which means stock A is higher rate of return than other stocks. This also represent that stock price is undervalued which means stock A has higher expected rate of return than the expectation of the investor (This undervalued price make the investor earn too much return). Therefore, the investor will invest more on A. This is the cause of the increasing price of stock A that simultaneously lowers the expected rate of return until it equals desired rate of return which is equilibrium.

In the other hand, stock B locates under SML line which means the expected rate of return is lower than required rate of return. This means that the stock price is overvalued and affects the investor to sell the stock which causes the decreasing of stock price and expected rate of return climbs higher simultaneously until it equals desired rate of return which is equilibrium.

So, stock A is undervalued and suggested to invest (which has more return) and stock B is overvalued with lower return which means that the investor should not buy.

3.1.3 Coefficient beta for decision making

Generally, stock market is divided into 2 period; bull market and bear market which defined by stock price changes or rate of return of the stock. Bear market is defined when the stock price indexes continuously decrease or the rate of return of the stock market is negative. Bull market is defined when the stock price indexes continuously increase or the rate of return of the stock market is positive.

According to the market classification, during bull market period, a stock with high value of the β earns higher return than other stock with lower value of β during bear market period, a stock with a lower value of β earns higher than other stocks with high value of β. When the rate of return of the stock market is positive, if the rate of return is lower than the rate of return without risk, a stock with low value of β earns more than a stock with high value of β.

Respectively, the new meaning of bull market and bear market can be defined. Bull market means the rate of return of the market is higher than the rate of return of the stock without risk ($E[R_m]-R_f$ is positive) and bear market means the rate of return of the market is lower than the rate of return of the stock without risk ($E[R_m]-R_f$ is negative).

From figure 3.2, the straight line represents the exchange between systematic risk and total stocks return (there are some stocks which their risk is a negative value or lower than stock without risk). Actually, this kind of stock does not exist but risk can be reduced from the theorem from figure 3.2, for a stock with its β is less than 1 called defensive security and a stock with its β is greater than 1 called aggressive security.

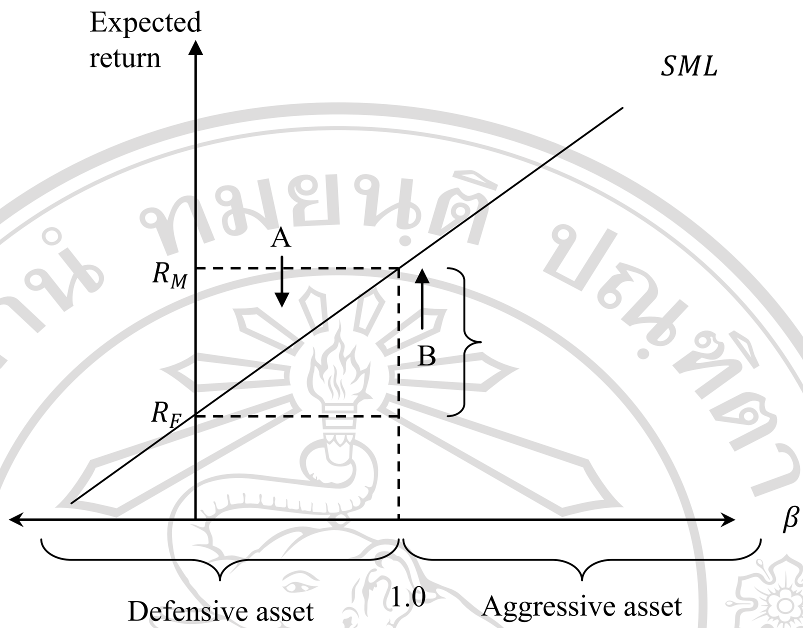

Figure 3.2: beta coefficient and Security Market Line

Source: Modify from Bodies, Kane and Marcus (2002: 273).

3.1.4 Unit Root Test[9]

Unit root test or the order of integration is widely used in economics applied in equation in order to test the stationary of the data. This study uses Dickey-Fuller testing in 2 methods as follows;

(1) Dickey Fuller Test (DF)

This methodology is used for testing the variable that moving over time. Autoregressive model, studied by Dickey and Fuller (1979), actually considers three different regression equations that can be used to test the presence of a unit root:

[9] See in Enders, W. (1948).

Pure random walk model

$$\Delta y_t = \gamma y_{t-1} + \varepsilon_t \tag{7}$$

Intercept term or drift term model

$$\Delta y_t = a_0 + \gamma y_{t-1} + \varepsilon_t \tag{8}$$

A drift and a linear time trend model

$$\Delta y_t = a_0 + \gamma y_{t-1} + a_2 t + \varepsilon_t \tag{9}$$

By

Δy_t	is lagged term of variable
a_0, γ, a_2	is constant term
t	is time trend
ε_t	is random variable has average equal zero and variance
	is fixed $\varepsilon_t \sim iid(0, \sigma_\varepsilon^2)$

The difference among three regressions concerns the presence of the deterministic element a_0 and $a_2 t$. The first is a pure random walk model, the second adds an intercept term or drift term, and the third includes both a drift and a linear time trend.

The parameter of interest in all the regressions equations is γ, if $\gamma \neq 0$, the $\{y_t\}$ sequence contains a unit root. The test involves estimating one (or more) of the equation above using OLS in order to obtain the estimated value of γ and the associated standard error. Comparing the resulting t-statistic with the appropriate value reported in the Dickey-Fuller tables allows us to determine whether accept or reject the null hypothesis equal zero. Thus, null hypothesis presented as follows

$H_0: \gamma = 0$ If acceptation $H_0: \gamma = 0$ means that y_t is stationary (reject unit root)

$H_1: \gamma < 0$ If acceptation (or rejection H_0) $H_1: \gamma < 0$ means that y_t is non-stationary (accept unit root)

(2) Augmented Dickey Fuller Test (ADF)

ADF test is applied on unit root testing that developed from DF test since DF test is not possible to test variable in case of variables have high serial correlation in error term. The equation can be written as follow;

Pure random walk model

$$\Delta y_t = \gamma y_{t-1} + \sum_{i=2}^{p} \beta_i \Delta y_{t-i+1} + \varepsilon_t \tag{10}$$

Intercept term or drift term model

$$\Delta y_t = a_0 + \gamma y_{t-1} + \sum_{i=2}^{p} \beta_i \Delta y_{t-i+1} + \varepsilon_t \tag{11}$$

A drift and a linear time trend model

$$\Delta y_t = a_0 + \gamma y_{t-1} + a_2 t + \sum_{i=2}^{p} \beta_i \Delta y_{t-i+1} + \varepsilon_t \qquad (12)$$

By lagged term *(p),* we can add a number of lagged terms until the data have no serial correlation in error term.

In Gamma value(γ) testing, t-statistic result can be compared with the appropriate value reported in the Augmented Dickey-Fuller tables which allows to determine whether to accept or reject the null hypothesis equal zero.

3.1.5 Bayesian Approach

In the classic approach, we determine the value of theta (θ) as fixed from a number of observations, but Bayesian approach specifies theta (θ) as the value of a random variable (Θ)

From Bayesian, the value observed $x = (x_1, x_2, \dots x_n)^T$ as set conditions on parameter value that the value $\theta = (\theta_1, \theta_2, \dots \theta_p)^T$ is unknown and will stay in a form of probability distribution $f(x|\theta)$, which come from what we fix θ is random variable (Θ). (Chen, 2008)

Set θ as a prior distribution that we will give the symbol of prior density of θ is $\pi(\theta)$, that is prior to its used to represent of density θ. Thus, if we collect data $X = x$, we will have a condition of density given $X = x$ which is called "posterior" and we can be written as $(x|\theta)$, and we have the prior distribution which is $\pi(\theta|\eta)$. η is determined to be vector of level second parameter (Hyper parameter), in the idea like Bayesian, we can estimate θ depend on the posterior distribution, which the format is;

$$p(\chi \mid x,\xi) = \frac{p(x,\chi \mid \xi)}{p(x \mid \xi)} = \frac{f(x \mid \chi)\phi(\chi \mid \xi)}{p(x \mid \xi)} \tag{13}$$

When $p(x \mid \xi) = \begin{cases} \sum_{\chi} f(x \mid \chi)\phi(\chi \mid \xi) & ,\chi \ \ is \ \ discreate \\ \int f(x \mid \chi)\phi(\chi \mid \xi)d\chi & ,\chi \ \ is \ \ continuous \end{cases}$

By the algebraic equation aforementioned , known as Bayes' theorem, there should be noticed that data which will exist two period that comes from the experiment or new data that received (which is shown in a form of the probability function) and the prior belief (which is shown in a form of the probability before the experiment ϕ) by the value begs for the hand of the part in the algebraic equation called that Marginal distribution of x when fix the vector of level second parameter is ξ and write get in a form of $m(x \mid \xi)$.(Cameron & Trivedi, 2005)

However, if we know the value of ξ , this variable will be departed from algebraic equation system. Because of that, we do not need to set the conditions on the constant, thus we will generate function form of the new distribution, by giving a simple format as;

$$p(\chi \mid x) = \frac{p(x,\chi)}{p(x)} = \frac{f(x \mid \chi)\phi(\chi)}{p(x)} \tag{14}$$

When $$p/x|\xi 0| \begin{cases} \sum_{\chi} f/x|\chi 0\phi/\chi 0 & ,\chi \ \textit{is discreate} \\ \int f/x|\chi 0\phi/\chi 0 d\chi & ,\chi \ \textit{is continuous} \end{cases}$$

We consider parameter value that has continual character from Bay's theory, then, distribution of $\chi | x$ can be generated which is called "Posterior distribution of χ" and we will give posterior distribution of χ as " $p/\chi | x0$" which it can be written as

$$p/\chi | x0| \frac{L/x|\chi 0\phi/\chi 0}{\int_N L/x|\chi 0\phi/\chi 0 d\chi} \tag{15}$$

$L/x|\chi 0$ or $f/x|\chi 0$ is a likelihood function and the symbol N represents parameter space of χ or support of $\phi/\chi 0$ which value is

$$p/x0| \int_N L/x|\chi 0\phi/\chi 0 d\chi \tag{16}$$

This is a normalizing constant of posterior distribution of χ and $p(x)$ is marginal probability distribution of x. For estimating problem of $p(x)$, there is no certain format of estimating like Bayesian of the value of χ that is basically base of posterior distribution of χ [$p/\chi | x0$]. The posterior estimator can be calculated a lot of varied data value such as average, median, mode, standard deviation and quantile.(Boyer & Kihlstrom, 1984)

For Bayesian's estimator, we can estimate value from data as follows;

Data	$x = (x_1, x_2, \ldots, x_n)'$
Parameter value	$\chi = (\chi_1, \chi_2, \ldots, \chi_p)'$
Likelihood	$L(x \mid \chi)$
Prior	$\phi(\chi)$

The estimation is based on Joint posterior that is

$$p(\chi \mid x) = \frac{L(x \mid \chi)\phi(\chi)}{\int L(x \mid \chi)\phi(\chi)\,d\chi} \qquad (17)$$

From equation (11), if the value of $p(x)$ is ignored which does not depend on χ and given x is constant. So, equation (11) can be rewritten in an easier format as;

$$p(\chi \mid x) \;\zeta\; f(x \mid \chi)\phi(\chi) \qquad (18)$$

Posterior ζ Likelihood × prior

Posterior probability is the ratio of probability multiplied by prior probability.

(Fhanjit Taemthong, 1994)

The value of ξ is uncertain but it can be written in the form of second-stage prior distribution levels or hyper parameter and rewrite equation with $h(\xi)$ which has posterior distribution of χ is

$$p(\chi \mid x) = \frac{p(x,\chi)}{p(x)} \tag{19}$$

$$p(\chi \mid x) = \frac{\int f(x,\chi,\xi)d\xi}{\iint f(x,\chi,\xi)d\xi d\chi}$$

$$p(\chi \mid x) = \frac{\int f(x \mid \chi)p(\chi \mid \xi)h(\xi)d\xi}{\iint f(x \mid \chi)p(\chi \mid \xi)h(\xi)d\xi d\chi} \tag{20}$$

We assume two situation is A and B, so the distribution conditional is (Anan Dejprom, 1996)

$$\Pr(A \mid B) = \frac{\Pr(A \cap B)}{\Pr(B)} = \frac{\Pr(B \mid A)\Pr(A)}{\Pr(B)} \tag{21}$$

3.1.6 Markov Chain Monte Carlo

Markov Chain Monte Carlo is the methodology of computationally intensive simulation for replacing integral method which was developed in 1980 by contributed possibility of opportunity to solve the real problem which is more complex.

The model of Markov chain is to contribute Markov process, which is stationary transition distribution, becomes the thing to specify, this is $p(\theta|x)$ and to generate the necessary part of model in the long run. Thus, the distribution of presented value of process will be certainly brought as the stationary transition distribution.

Thus, the instruction of Markov chain simulation is applied to find the distribution of $p(\theta|x)$ which is called "Markov Chain Monte Carlo method (MCMC)".

3.1.7 Gibbs Sampler

We begin with the Gibbs sampler, a member of MCMC class that it is easy to describe and imply.

Let $\chi = (\chi_1 \quad \chi_2)'$ have posterior density $p(\chi) = p(\chi_1,\chi_2)$, whereas notational simplicity we suppress dependence on *y*. If the conditional densities are known, which is not guaranteed as knowledge of both $p(\chi_1|\chi_2)$ and $p(\chi_2|\chi_1)$ is necessary, which alternating sequential draws from $p(\chi_1|\chi_2)$ and $p(\chi_2|\chi_1)$ in the limit converge to draws from $p(\chi_1|\chi_2)$. For instance, we consider a situation that has 3 parameters (χ_1,χ_2,χ_3) that is three conditions of posterior distribution, when we make the first iterative data, we will get data from.

$$f_1(\chi_1|\chi_2,\chi_3,y)$$

$$f_2(\chi_2|\chi_3,\chi_1,y)$$

$$f_3(\chi_3|\chi_1,\chi_2,y) \qquad \text{By } y = (y_1,y_2,\ldots,y_n)'$$

So we will get process of Gibbs sampling as following;

1. Consider arbitrary set of starting parameter value, which we get $\chi_{1,0},\chi_{2,0},\chi_{3,0}$

2. Generate M+N set of random number by iterative data set from full conditional posterior distribution, we will have general form of rank (*i-th*) as $(\chi_{1,i}, \chi_{2,i}, \chi_{3,i})$ so, we get

(a) $\chi_{1,i+1}$ From $f_1(\chi_1 \mid \chi_{2,i}, \chi_{3,i}, y)$

(b) $\chi_{2,i+1}$ From $f_2(\chi_2 \mid \chi_{3,i}, \chi_{1,i+1}, y)$ and

(C) $\chi_{3,i+1}$ From $f_3(\chi_3 \mid \chi_{1,i+1}, \chi_{2,i+1}, y)$ from (i+1)-*th* realization rank

3. Omit M that we got from the second combination process and use the last value of N to generate random sample $\{(\chi_{1,i}, \chi_{2,i}, \chi_{3,i})\}_{i=M+1}^{M+N}$, and estimate posterior marginal by random sample.

Beneath the easy condition, Geman & Geman (1984) have shown joint distribution of above random sample, which will be converge exponentially to joint posterior distribution of (χ_1, χ_2, χ_3), before we do realization form of random sample from joint distribution, we will estimate data set from

$$\hat{\chi}_i = \frac{1}{N-M} \sum_{j=M+1}^{N} \chi_{i,j} \tag{22}$$

$$\hat{\omega}^2_i = \frac{1}{N-M-1} \sum_{j=M+1}^{N} (\chi_{i,j} - \hat{\chi}_i)^2 \tag{23}$$

Credible interval value (l, u) of χ_i must have under condition, for example $p(l \le \chi_i \le u) = 0.95$ etc.

This is algorithm that will be replaced value to each possible state, for finding the best solution, which in this case is the value at risk (β) and Probability Distribution Function (PDF). (Carter & Kohn, 1994)

The mentioned results do not tell us how many cycles are needed for convergence, which is model dependent. Varieties of diagnostic tests of convergence are available. Because estimates of posterior moments should be based on draws from the posterior distribution, it is standard practice to discard the earlier results from the chain, the so-called **burn-in phrase.**[10]

3.1.8 Bayes Factor

From statistics, we use Bayes factors to be the choice of Bayesian approach to test classical hypothesis that is data specification beneath hypothesis, by using a problem of model filtration between the model M_i and M_j on data base of vector y by M_i and M_j be like assumption that goes up, which each the model will be probability of itself, such as $p(y \mid M_i)$ or $p(y \mid M_j)$(Taemthong, 1994). So, we can fix prior probability as $p(M_i)$ and $p(M_i) = 1 - p(M_j)$ then we get posterior probability as $p(M_i \mid y)$ and $p(M_i \mid y) = 1 - p(M_j \mid y)$ because of the prior belief will modify the data to become posterior belief later. Thus, we can modify data to be odds ratio which is

$$Odds = \frac{Probability}{1 - Probability} \tag{24}$$

[10] See Microeconometrics, methods and Applications. A. Colin Cameron and Pravin K. Trivedi. and Kass and Raftery(1995) for a review of Bayes factors.

Thus, Bayes factor: BF is

$$BF_{ij} = \frac{p(y|M_i)}{p(y|M_j)} \qquad (25)$$

It means

Posterior odds = Bayes factor × prior odds

Bayes factor is ratio between posterior odds of M_i and prior odds.

By

$$p(y|M_i) = \int p(x|\chi_i, M_i)\phi(\chi_i|M_i)d\chi_i \qquad (26)$$

By χ_i is the vector of the parameter beneath M_i

$\phi(\chi_i|M_i)$ is prior distribution of χ_i

$p(y|\chi_i, M_i)$ is probability of y when we fix χ_i or they should be of χ

So, $p(y|M_i)$ is called Marginal likelihood[11] for i model, generally model M_i and M_j are parameterized by vector of parameters χ_i and χ_j thus Bayes factor is

$$BF_{ij} = \frac{p(y|M_i)}{p(y|M_j)} \qquad (25)$$

[11] It is straightforward to show that a Bayes factor is the ratio of posterior to prior odds.

$$= \frac{\int p(\chi_i \mid M_i) p(y \mid \chi_i, M_i) d\chi_i}{\int p(\chi_j \mid M_j) p(y \mid \chi_j, M_j) d\chi_j} \qquad (27)$$

If we get $p(M_i)$ is prior probability of the M_i model.

$$BF_{ij} = \frac{p(M_i \mid y)}{p(M_j \mid y)} \div \frac{p(M_i)}{p(M_j)} \qquad (28)$$

This equation is posterior odds in favor of the M_i model, which was separated by posterior odds in favor of the M_j model. (Cameron & Trivedi, 2005)

The marginal likelihood is simply the normalizing constant of the posterior density. Suppressing the model index M_j for the simplicity, the marginal likelihood can be written as

$$p(y) = \frac{f(y \mid \chi) \phi(\chi)}{\phi(\chi \mid y)} \qquad (29)$$

Where $f(y \mid \chi)$ is the likelihood, $\phi(\chi)$ the prior density, $\phi(\chi \mid y)$ the posterior density, and χ is evaluated at the posterior mean estimate χ. The posterior density $\phi(\chi \mid y)$ is computed using the technique of reduced conditional MCMC run of **Chib(1995)**[12].

[12] See in "MCMC Method" in Durlauf, S. N., & Blume, L. E. (2008).

3.1.9 Ordinary Least Square

The method of least squares or ordinary least squares (OLS) is used to solve the determined systems. Least squares are often applied in statistical contexts, particularly regression analysis. Least squares can be interpreted as a method of fitting data. The best fit in the least-squares sense is that instance of the model for which the sum of squared residuals has the least value which is the difference between an observed value and the value given by the model. The method was first described by Carl Friedrich Gauss in 1794. Least squares correspond to the maximum likelihood criterion if the experimental errors have a normal distribution and can also be derived as a method of moment's estimator.

Let

$$Y = X\beta + u \quad (30)$$

By

Y is dependent variable

X is matrix of independent variables

β is coefficients of the fit

u is error term

Let $\hat{\beta}$ defined as a vector which k is a component the definition of error term or residuals term in vector form is

$$e = y - x\hat{\beta} \quad (31)$$

By

y is dependent variable

x is matrix of independent variables

β is computed coefficients

e is computed residual

As theory defined, least sum square residuals $(e'e)$ is the method used for the selection of the $\hat{\beta}$, so the equation can rewrite in pattern form of $y, x, \hat{\beta}$ as

$$e'e = (y - x\hat{\beta})'(y - x\hat{\beta}) \quad (32)$$

$$= y'y - 2\hat{\beta}'x'y + \hat{\beta}'x'x\hat{\beta} \quad (33)$$

Thus

$$\partial(e'e)/\partial\hat{\beta} = -2x'y + 2x'x\hat{\beta} \quad (34)$$

we defined this equation equal zero and this equation can be rewritten in order to estimate $\hat{\beta}$ as follow

$$(x'x)\hat{\beta} = x'y \quad (35)$$

From equation (35), it is called OLS normal equations and expresses the nonsingular of $x'x$ from

$$\hat{\beta} = (x'x)^{-1}(x'y)^{13} \tag{36}$$

3.2 Methodology

There are three parts in this research. In the first section, we use a Bayesian approach for CAPM to estimate beta (β) and find standard error (S.E.) of this beta. In the second section, we use ordinary least square (OLS) for CAPM to estimate beta (β) and find the standard error (S.E.) of this beta (β) from OLS. In the last section, we compare the standard error (S.E.) from the Bayesian approach and ordinary least square (OLS) to identify the best method to determine risk and rate of return.

The first section, data collection and economic variable calculation to analysis CAPM equation by **CAPM theory**[14] can be applied for this research by each industry group index is determined from SET index of Thai economy by using a Bayesian approach for CAPM to estimate beta (β) and find standard error (S.E.) of this beta.

Let *AGRI* is agriculture industry and food industry group index

CONS is consumer Products group index

FIN is financials group index

INDUS is manufacture group index

[13] See Songsak Sriboonchitta. (2004). *Econometric theory and application*. Chiang Mai.

[14] $E(R_i) = R_f + (E(R_m) - R_f)\beta_i$, From equation(2)

PROP	is property and construction group index
RES	is resources group index
SER	is services group index
TECH	is technology group index

Let expected return of each asset group index is denoted

RAGRI	is the expected return on agriculture-industry and food-industry group index
RCONS	is the expected return on consumer Products group index
RFIN	is the expected return on finance group index
RINDUS	is the expected return on manufacture group index
RPROP	is the expected return on property and construction group index
RRES	is the expected return on resources group index
RSER	is the expected return on services group index
RTECH	is the expected return on technology group index

Asset return equation is

$$RP_{i,t} = \left(ln(p_{i,t}) - ln(p_{i,t-1})\right) \times 100 \quad (37)$$

By

i	is asset group index (Agri, Cons, Fin, Indus, Prop, Res, Ser, Tech)
$RP_{i,t}$	is the expected return on the asset group index i at time t
ln	is natural logarithm

$p_{i,t}$ is asset group index *i* at time *t*

$p_{i,t-1}$ is asset group index *i* at time *t-1*

For example: Agriculture industry and Food Industry have return equation as followed:

$$RAGRI_t = \left(ln(AGRI_t) - ln(AGRI_{t-1})\right) \times 100 \quad (38)$$

By $RAGRI_{i,t}$ is the expected return on the agriculture industry and food industry group index *i* at time *t*

ln is natural logarithm

$AGRI_{i,t}$ is agriculture industry and food industry group index *i* at time *t*

$RAGRI_{i,t-1}$ is agriculture industry and food industry group index *i* at time *t-1*

Market return equation is

$$RThi_t = \left(ln(THI_t) - ln(THI_{t-1})\right) \times 100 \quad (39)$$

By $RThi_t$ is the market return on the Stock Exchange Thailand index of Thai economy at time *t*

ln is natural logarithm

THI_t is Stock Exchange Thailand index of Thai economy at time *t*

THI_{t-1} is Stock Exchange Thailand index of Thai economy at time *t-1*

Calculate risk-free rate return equation from

$$RP_{i,t} = \beta_0 + \beta_1 \boldsymbol{RTHI_t}^{15} + \varepsilon_t \tag{40}$$

By: $RP_{i,t}$ is the expected return on the asset group index *i* at time *t*

$\boldsymbol{RTHI_t}$ is return of Stock Exchange Thailand index of Thai economy at time *t*

β_0 is the return of asset that is higher or lower than market's return

β_1 is the investment's risk in the asset group index at time *t*

ε_t is error term at time *t*

For example: Agriculture industry and Food Industry have equation as followed:

$$\boldsymbol{RAGRI_t}^{16} = \beta_0 + \beta_1 \boldsymbol{RTHI_t}^{17} + \varepsilon_t \tag{41}$$

By: $RAGRI_t$ is the expected return on the agriculture industry and food industry group index at time *t*

$RTHI_t$ is return of Stock Exchange Thailand index of Thai economy at time *t*

[15] From equation (39), $RThi_t = \left(ln(THI_t) - ln(THI_{t-1})\right) \times 100$

[16] From equation (38), $RAGRI_t = \left(ln(AGRI_t) - ln(AGRI_{t-1})\right) \times 100$

[17] From equation (39), $RTHI_t = \left(ln(THI_t) - ln(THI_{t-1})\right) \times 100$

β_0 is the return of asset that is higher or lower than market's return.

β_1 is the investment's risk in the agriculture industry and food industry group index at time t .

ε_t is error term at time t.

The second section, the analysis CAPM equation each industry group index is determined from SET index of Thai economy by using a Ordinary Least Square (OLS) for CAPM to estimate beta (β) and its standard error (S.E.) .

The last section, we compare the standard error (S.E.) from the Bayesian approach and ordinary least square (OLS) to identify the best method for determine risk and rate of return when investors are about to invest for each Thailand asset group index.