Chapter 2

Methodology and Model Specifications

This dissertation employs the univariate and multivariate GARCH models to estimate and forecast volatility and volatility spillovers with symmetric and asymmetric effects in financial markets. First, the returns of market i at time t are calculated as follows:

$$R_{i,t} = \log(P_{i,t} / P_{i,t-1}) \tag{2.1}$$

where $P_{i,t}$ and $P_{i,t-1}$ are the closing prices of market i for days *t* and *t-1*, respectively.

Second, stationary of the data are tested by using the Augmented Dickey-Fuller (ADF) test, which is given as follows:

$$\Delta y_t = \alpha + \beta t + \theta y_{t-1} + \sum_{i=1}^{p} \phi_i \Delta y_{t-i} + \varepsilon_t \tag{2.2}$$

The null hypothesis is $\theta = 0$ which, if rejected, means that the series y_t is stationary.

Third, a wide range of conditional volatility models have been used to estimate and forecast volatility and volatility spillovers with symmetric and asymmetric effects in financial markets. Univariate and multivariate conditional volatility models, namely GARCH, GJR, EGARCH, CCC, DCC, VARMA-GARCH and VARMA-AGARCH, are used in this dissertation to capture the volatility in financial markets in South-East Asian countries.

2.1 GARCH

Engle (1982) introduced the Autoregressive Conditional Heteroskedasticity (ARCH) model that volatility is affected symmetrically by positive and negative shocks of equal magnitude from previous periods. Bollerslev (1986) generalized ARCH(r) to the GARCH(r,s) model, as follows:

$$h_t = \omega + \sum_{i=1}^{r} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{s} \beta_j h_{t-j} \quad (2.3)$$

where $\omega > 0$, $\alpha_i \geq 0$ for i = 1,…,r, and $\beta_j \geq 0$ for j = 1,…,s, are sufficient to ensure that the conditional variance, $h_t > 0$. The α_i represent the ARCH effects and β_j represent the GARCH effects.

GARCH(r,s) shows that the volatility is not only effected by shocks but also by its own past. The model also assumes positive shocks ($\varepsilon_t > 0$) and negative shocks ($\varepsilon_t < 0$) of equal magnitude have the same impact on the conditional variance.

2.2 GJR

In order to accommodate differential impacts on the conditional variance of positive and negative shocks of equal magnitude, Glosten et al. (1993) proposed the following specification for h_t:

$$h_t = \omega + \sum_{i=1}^{r} \left(\alpha_i + \gamma_i I(\varepsilon_{t-i}) \right) \varepsilon_{t-i}^2 + \sum_{j=1}^{s} \beta_j h_{t-j} \quad (2.4)$$

where $I(\varepsilon_{t-i})$ is an indicator function that takes the value 1 if $\varepsilon_{t-i} < 0$ and 0 otherwise. The impact of positive shocks and negative shocks on conditional variance allows for an asymmetric impact. The expected value of γ_i is positive, such that negative shocks have a higher impact on volatility than do positive shocks of equal magnitude. It is not possible for leverage to be present in the GJR model, whereby negative shocks increase volatility and positive shocks of equal magnitude decrease volatility.

If r = s = 1, $\omega > 0$, $\alpha_1 \geq 0$, $\alpha_1 + \gamma_1 \geq 0$ and $\beta_1 \geq 0$ are sufficient conditions to ensure that the conditional variance h_t > 0. The short run persistence of positive (negative) shocks is given by α_1 $(\alpha_1 + \gamma_1)$. When the conditional shocks, η_t, follow a symmetric distribution, the short run persistence is $\alpha_1 + \gamma_1 / 2$, and the contribution of shocks to long run persistence is $\alpha_1 + \gamma_1 / 2 + \beta_1$.

2.3 EGARCH

Nelson (1991) proposed the Exponential GARCH (EGARCH) model, which incorporates asymmetries between positive and negative shocks on conditional volatility. The EGARCH model is given by:

$$\log h_t = \omega + \sum_{i=1}^{r} \alpha_i \left| \eta_{t-i} \right| + \sum_{i=1}^{r} \gamma_i \eta_{t-i} + \sum_{j=1}^{s} \beta_j \log h_{t-j} \quad (2.5)$$

In equation (2.5), $|\eta_{t-i}|$ and η_{t-i} capture the size and sign effects, respectively, of the standardized shocks. If γ_i is less than zero, positive shocks will have a smaller effect on volatility than will negative shocks of equal magnitude. Moreover, (2.5) can allow for asymmetric and leverage effects if $\gamma < 0$ and $\gamma < \alpha < -\gamma$ exist. As EGARCH uses the logarithm of conditional volatility, there are no restrictions on the parameters in (2.5). As the standardized shocks are assumed to have finite moments, the moment conditions of (2.5) are entirely straightforward.

Lee and Hansen (1994) derived the log-moment condition for GARCH(1,1) as

$$E(\log(\alpha_1\eta_t^2 + \beta_1)) < 0 \tag{2.6}$$

This is important in deriving the statistical properties of the QMLE. McAleer et al. (2007) established the log-moment condition for GJR(1,1) as

$$E(\log((\alpha_1 + \gamma_1 I(\eta_t))\eta_t^2 + \beta_1)) < 0 \tag{2.7}$$

The respective log-moment conditions can be satisfied even when $\alpha_1 + \beta_1 < 1$ (that is, in the absence of second moments of the unconditional shocks of the GARCH(1,1) model), and when $\alpha_1 + \gamma/2 + \beta_1 < 1$ (that is, in the absence of second moments of the unconditional shocks of the GJR(1,1) model).

2.4 VARMA-GARCH

The VARMA-GARCH model of Ling and McAleer (2003) assumes symmetry in the effects of positive and negative shocks of equal magnitude on conditional volatility. Let the vector of returns on m (≥ 2) financial assets be given by:

$$Y_t = E(Y_t \mid F_{t-1}) + \varepsilon_t \tag{2.8}$$

$$\varepsilon_t = D_t \eta_t \tag{2.9}$$

$$H_t = \omega + \sum_{k=1}^{r} A_k \vec{\varepsilon}_{t-k} + \sum_{l=1}^{s} B_l H_{t-l} \tag{2.10}$$

where $H_t = (h_{1t},...,h_{mt})'$, $\omega = (\omega_1,...,\omega_m)'$, $D_t = diag(h_{i,t}^{1/2})$, $\eta_t = (\eta_{1t},...,\eta_{mt})'$, $\vec{\varepsilon}_t = (\varepsilon_{1t}^2,...,\varepsilon_{mt}^2)'$, A_k and B_l are $m \times m$ matrices with typical elements α_{ij} and β_{ij}, respectively, for $i,j = 1,\ldots,m$, $I(\eta_t) = diag(I(\eta_{it}))$ is an $m \times m$ matrix, and F_t is the past information available to time t. Spillover effects are given in the conditional volatility for each asset in the portfolio, specifically where A_k and B_l are not diagonal matrices. For the VARMA-GARCH model, the matrix of conditional correlations is given by $E(\eta_t \eta_t') = \Gamma$.

2.5 VARMA-AGARCH

An extension of the VARMA-GARCH model is the VARMA-AGARCH model of McAleer et al. (2009), which assumes asymmetric impacts of positive and negative shocks of equal magnitude, and is given by

$$H_t = \omega + \sum_{k=1}^{r} A_k \vec{\varepsilon}_{t-k} + \sum_{k=1}^{r} C_k I_{t-k} \vec{\varepsilon}_{t-k} + \sum_{l=1}^{s} B_l H_{t-l} \qquad (2.11)$$

where C_k are $m \times m$ matrices for $k = 1,\ldots,r$ and I_t = diag($I_{1t},\ldots,I_{mt}$), so that

$$I = \begin{cases} 0, \varepsilon_{k,t} > 0 \\ 1, \varepsilon_{k,t} \leq 0. \end{cases}$$

From equation (2.11), if m = 1, the model reduces to the asymmetric univariate GARCH, or GJR. If C_k = 0 for all k, the model reduces to VARMA-GARCH.

2.6 CCC

If the model given by equation (2.11) is restricted so that C_k = 0 for all k, with A_k and B_l being diagonal matrices for all k, l, then VARMA-AGARCH reduces to

$$h_{it} = \omega_i + \sum_{k=1}^{r} \alpha_i \varepsilon_{i,t-k} + \sum_{l=1}^{s} \beta_i h_{i,t-l} \qquad (2.12)$$

which is the constant conditional correlation (CCC) model of Bollerslev (1990), for which the matrix of conditional correlations is given by $E(\eta_t \eta_t') = \Gamma$. As given in equation (2.12), the CCC model does not have volatility spillover effects across different financial assets, and does not allow conditional correlation coefficients of the returns to vary over time.

2.7 DCC

Engle (2002) proposed the Dynamic Conditional Correlation (DCC) model, which allows for two-stage estimation of the conditional covariance matrix. In the first stage, univariate volatility models are estimated to obtain the conditional volatility, h_t, of each asset. At the second stage, asset returns are transformed by the estimated standard deviations from the first stage, and are then used to estimate the parameters of DCC. The DCC model can be written as:

$$y_t \mid F_{t-1} \sim (0, Q_t), \quad t = 1, \ldots, T, \tag{2.13}$$

$$Q_t = D_t \Gamma_t D_t, \tag{2.14}$$

where $D_t = diag(h_{1t}^{1/2}, \ldots, h_{mt}^{1/2})$ is a diagonal matrix of conditional variances, with m asset returns, and F_t is the information set available at time t. The conditional variance is assumed to follow a univariate GARCH model, as follows:

$$h_{it} = \omega_i + \sum_{k=1}^{r} \alpha_{i,k} \varepsilon_{i,t-k} + \sum_{l=1}^{s} \beta_{i,l} h_{i,t-l} \tag{2.15}$$

when the univariate volatility models have been estimated, the standardized residuals, $\eta_{it} = y_{it} / \sqrt{h_{it}}$, are used to estimate the dynamic conditional correlations, as follows:

$$Q_t = (1 - \phi_1 - \phi_2) S + \phi_1 \eta_{t-1} \eta'_{t-1} + \phi_2 Q_{t-1} \tag{2.16}$$

$$\Gamma_t = \left\{ (diag(Q_t)^{-1/2} \right\} Q_t \left\{ (diag(Q_t)^{-1/2} \right\}, \tag{2.17}$$

where S is the unconditional correlation matrix of the returns shocks, and equation (2.17) is used to standardize the matrix estimated in (2.16) to satisfy the definition of a correlation matrix.