Chapter 3

Comparing the Volatility Index and Index of Volatility for Europe and the USA

This chapter construct indexes of volatility for the USA, and Europe by two ways are single index model consisting of univariate volatility model (ARCH, GARCH, GJR, EGARCH, and Riskmetrics^{TM}) of portfolio return and a portfolio model which use multivariate volatility model (CCC, VARMA-GARCH, VARMA-AGARCH, and DCC) to forecast variance and covariance to compute portfolio risk. Then compare indexes of volatility and volatility index (VIX for the USA and VSTOXX for Europe) by using the predictive power of Value-at-Risk. Finally, this chapter finds out correlations of Value-at-Risk forecast calculated from various models. This chapter is a revised version from the original paper of Kunsuda Ninanussornkul, Michael McAleer, and Songsak Sriboonchitta; presented at the Second Conference of The Thailand Econometric Society, Chiang Mai, Thailand in Appendix A in 5 – 6 January 2009.

Abstract

Volatility forecasting is an important task in financial markets. In 1993, the Chicago Board Options Exchange (CBOE) introduced the CBOE volatility index, VIX, and it quickly became the benchmark for stock market volatility. After 2003, the CBOE reported a new VIX, and changed the original VIX to VXO. The new VIX estimates reflect expected volatility from the prices of stock index options for a wide range of strike prices, not just at-the money strikes, as in the original VIX, so that the model-free implied volatility is more likely to be informationally efficient than the Black-Scholes implied volatility. However, the new VIX uses the model-free implied volatility, which is not based on a specific volatility model. This paper constructs an index of volatility for Europe and the USA by using a single index model or the covariance matrix of the portfolio forecast the variance of a portfolio. Using univariate and multivariate conditional volatility models. A comparison between the volatility index and the index of volatility using predictive power of Value-at-Risk will be made to determine the practical usefulness of these indexes.

Keywords: Index of volatility, volatility index, single index, portfolio model, Value-at-Risk

3.1 Introduction

Volatility forecasting is an important task in financial markets, and it has held the attention of academics and practitioners over the last two decades. Academics are interested in studying temporal patterns in expected returns and risk. For practitioners, volatility has an importance in investment, security valuation, risk management, and monetary policy making. Volatility is interpreted as uncertainty. It becomes a key factor to many investment decisions and portfolio creations because investors and portfolio managers want to know certain levels of risk. Volatility is also the most important variable in the pricing of derivative securities. (see Fleming, J., Ostdiek, B. and Whaley, R.E.(1995) and Poon, S. and Granger, C.W.J.(2003))

Volatility has an effect on financial risk management exercise for many financial institutions around the world since the first Basle Accord was established in 1996. It is an important ingredient to calculate Value-at-Risk (VaR). Value-at-Risk may be defined as "a worst case scenario on a typical day". If a financial institution's VaR forecasts are violated more than can reasonably be expected, given the confidence level, the financial institution will hold a higher level of capital. (McAleer, M. (2008a))

In 1993, the Chicago Board Options Exchange (CBOE) introduced the CBOE volatility index, VIX, and it quickly became the benchmark for stock market volatility. After 2003, the CBOE reported a new VIX, and changed the original VIX to VXO. The new VIX estimates reflect expected volatility from the prices of stock index options for a wide range of strike prices, not just at-the-money strikes, as in the original VIX. Therefore, the model-free implied volatility is more likely to be informationally efficient than the Black-Scholes implied volatility. However, the new

VIX uses the model-free implied volatility, which is not based on a specific volatility model. (See, Jiang, G.J. and Tian, Y.S. (2005))

In Europe, there is also a volatility index. Its calculation method is the same method as CBOE's. One type of volatility indices in Europe is the VSTOXX volatility index, which was introduced on 20 April 2005. It has provided a key measure market expectations of near-term volatility based on the Dow Jones EURO STOXX 50 options prices.

Most studies in the literature about construction and prediction the volatility index. (See Skiadopoulos, G.S.(2004) Moraux, F., Navatte, P. and Villa, C. (1999) and Fernades, M. and Medeiros, M.C.)

This paper would like to construct an index of volatility by using conditional volatility models by: (1) fitting a univariate volatility model to the portfolio returns (hereafter called the single index model (see McAleer, M. and da Veiga, B. (2008a,2008b)), and (2) using a multivariate volatility model to forecast the conditional variance of each asset in the portfolio as well as the conditional correlations between all asset pairs in order to calculate the forecasted portfolio variance (hereafter called the portfolio model) for the USA and Europe. Then, comparison between the index of volatility and the volatility index will be made by using the predictive power of Value-at-Risk.

The organization of the paper is as follows: section 3.2 presents the index of volatility and section 3.3 shows volatility index. The data and estimation are in Section 3.4. Empirical results, Value-at-Risk, and conclusion are in Section 3.5, 3.6, and 3.7, listed respectively.

3.2 Index of Volatility

This paper uses the price sector indices of S&P 500 for the USA and STOXX for Europe. There are 10 sector indices, however this paper aggregates price sector indices to be 3 sectors by using market capitalization as a weighted variable. For example, if we would like to aggregate sector *1*, *2*, *3* together, the model is as follows:

$$P_{123t} = \frac{MV_{1t} \times P_{1t} + MV_{2t} \times P_{2t} + MV_{3t} \times P_{3t}}{MV_{1t} + MV_{2t} + MV_{3t}} \quad (3.1)$$

where P_{123t} is the aggregate price sector index of sector *1,2,* and *3*, MV_{it} is market capitalization of sector i (i = 1, 2, 3), and P_{it} is price sector index of sector i (i = 1, 2, 3).

Then we compute returns of each sector as follows:

$$R_{i,t} = 100 \times \log(P_{i,t} / P_{i,t-1}) \quad (3.2)$$

where $P_{i,t}$ and $P_{i,t-1}$ are the closing prices of sector i (i = 1, 2, 3) at days t and t-*1*, Then we construct index of volatility by two model follows:

3.2.1 Single index model

This paper constructs a single index model following these steps:

(1) Compute portfolio returns by using market capitalization at the first day as a weighted variable, as follows:

$$Port_t = \frac{MV_1 \times r_{1t} + MV_2 \times r_{2t} + MV_3 \times r_{3t}}{MV_1 + MV_2 + MV_3} \tag{3.3}$$

where $Port_t$ is portfolio returns, MV_i is market capitalization of sector i (i = 1, 2, 3), and r_{it} is returns of sector i (i = 1, 2, 3).

(2) Estimating univariate volatility of portfolio returns from the first step by mean equation have constant term and autoregressive term (AR(1)) in all models. The univariate volatility is the index of volatility. Moreover, this paper computes Riskmetrics™ by using the exponentially weighted moving average model (EWMA) of portfolio returns.

Univariate Volatility

ARCH

Engle, R.F. (1982) proposed the Autoregressive Conditional Heteroskedasticity of order p, or ARCH(p), follows:

$$h_t = \omega + \sum_{j=1}^{p} \alpha_j \varepsilon_{t-j}^2 \tag{3.4}$$

where $\omega > 0$ *and* $\alpha_j \geq 0$

GARCH

Bollerslev, T. (1986) generalized ARCH(p) to the GARCH(p,q), model as follows:

$$h_t = \omega + \sum_{j=1}^{p} \alpha_j \varepsilon_{t-j}^2 + \sum_{i=1}^{q} \beta_i h_{t-i} \qquad (3.5)$$

where $\omega > 0$, $\alpha_j \geq 0$ for j = 1,…,p and $\beta_i \geq 0$ for i = 1,…,q are sufficient to ensure that the conditional variance $h_t > 0$.

The model also assumes positive shock ($\varepsilon_t > 0$) and negative shock ($\varepsilon_t < 0$) of equal magnitude have the same impact on the conditional variance.

GJR

Glosten, L.R., et al. (1993) accommodate differential impact on the conditional variance of positive and negative shocks of equal magnitude. The GJR(p,q) model is given by:

$$h_t = \omega + \sum_{j=1}^{p} \left(\alpha_j + \gamma_j I(\varepsilon_{t-j})\right) \varepsilon_{t-j}^2 + \sum_{i=1}^{q} \beta_i h_{t-i} \qquad (3.6)$$

where the indicator variable, $I(\varepsilon_t)$, is defined as: $I(\varepsilon_t) = \begin{cases} 1, \varepsilon_t \leq 0 \\ 0, \varepsilon_t > 0 \end{cases}$. If p = q = 1, $\omega > 0$, $\alpha_1 \geq 0$, $\alpha_1 + \gamma_1 \geq 0$, and $\beta_1 \geq 0$ then it has sufficient conditions to ensure that the conditional variance $h_t > 0$. The short run persistence of positive (negative) shocks is given by $\alpha_1 \left(\alpha_1 + \gamma_1\right)$. When the conditional shocks, η_t, follow a symmetric distribution, the short run persistence is $\alpha_1 + \gamma_1 / 2$, and the contribution of shocks to long run persistence is $\alpha_1 + \gamma_1 / 2 + \beta_1$.

EGARCH

Nelson, D. (1991) proposed the Exponential GARCH (EGARCH) model, which incorporates asymmetries between positive and negative shocks on conditional volatility. The EGARCH model is given by:

$$\log h_t = \omega + \sum_{i=1}^{p} \alpha_i \left|\eta_{t-i}\right| + \sum_{i=1}^{p} \gamma_i \eta_{t-i} \sum_{j=1}^{q} \beta_j \log h_{t-j} \qquad (3.7)$$

In equation (3.7), $\left|\eta_{t-i}\right|$ and η_{t-i} capture the size and sign effects, respectively, of the standardized shocks. EGARCH in (3.7) uses the standardized residuals. As EGARCH uses the logarithm of conditional volatility, there are no restrictions on the parameters in (3.7). As the standardized shocks have finite moments, the moment conditions of (3.7) are straightforward.

Lee, S.W. and Hansen, B.E. (1994) derived the log-moment condition for GARCH (1,1) as

$$E(\log(\alpha_1 \eta_t^2 + \beta_1)) < 0 \qquad (3.8)$$

This is important in deriving the statistical properties of the QMLE. McAleer, M., et al. (2007) established the log-moment condition for GJR(1,1) as

$$E(\log((\alpha_1 + \gamma_1 I(\eta_t))\eta_t^2 + \beta_1)) < 0 \qquad (3.9)$$

The respective log-moment conditions can be satisfied even when $\alpha_1 + \beta_1 > 1$(that is, in the absence of second moments of the unconditional shocks of the GARCH(1,1) model) and when $\alpha_1 + \gamma / 2 + \beta_1 < 1$(that is, in the absence of second moments of the unconditional shocks of the GJR(1,1) model).

Riskmetrics™

Riskmetrics™ (1996) developed a model which estimates the conditional variances and covariances based on the exponentially weighted moving average (EWMA) method, which is, in effect, a restricted version of the ARCH(∞) model. This approach forecasts the conditional variance at time t as a linear combination of lagged conditional variance and the squared unconditional shock at time *t-1*. The Riskmetrics™ model estimate the conditional variances follow:

$$h_t = \lambda h_{t-1} + (1-\lambda)\varepsilon_{t-1}^2 \quad (3.10)$$

where λ is a decay parameter. Riskmetrics™ (1996) suggests that λ should be set at 0.94 for purposes of analyzing daily data.

3.2.2 Portfolio model

This paper constructs the portfolio model by following these steps:

(1) Estimate multivariate volatility of three sectors for Europe and the USA by mean equation so that they have constant term and autoregressive term (AR(1)) in all models. Then compute variance and covariance matrix.

(2) Compute index of volatility by using market capitalization at the first observation is a weighted variable. This paper has three sectors so that we have the three conditional variances and three covariance estimated. It follows that:

$$IVol_t = \lambda_1^2 h_{1t} + \lambda_2^2 h_{2t} + \lambda_3^2 h_{3t} + 2\lambda_1\lambda_2 h_{12t} + 2\lambda_1\lambda_3 h_{13t} + 2\lambda_2\lambda_3 h_{23t} \quad (3.11)$$

where $IVol_t$ is index of volatility, h_{it} is conditional variances of sector i (i=1,2,3), h_{ijt} is covariance of sector i (i=1,2,3), and $\lambda_1 = \frac{MV_1}{MV_1 + MV_2 + MV_3}$, $\lambda_2 = \frac{MV_2}{MV_1 + MV_2 + MV_3}$, and $\lambda_3 = \frac{MV_3}{MV_1 + MV_2 + MV_3}$.

The number of covariance increases dramatically with m, the number of assets in the portfolio. Thus, for m = 2, 3, 4, 5, 10, 20, the number of covariance is 1, 3, 6, 10, 45, 190, respectively. This increases the computation burden significantly. (See details in McAleer, M. (2008a))

Multivariate volatility

VARMA-GARCH

The VARMA-GARCH model of Ling, S. and McAleer, M. (2003), assumes symmetry in the effects of positive and negative shocks of equal magnitude on conditional volatility. Let the vector of returns on m (≥ 2) financial assets be given by:

$$Y_t = E(Y_t \mid F_{t-1}) + \varepsilon_t \quad (3.12)$$

$$\varepsilon_t = D_t \eta_t \quad (3.13)$$

$$H_t = \omega + \sum_{k=1}^{p} A_k \vec{\varepsilon}_{t-k} + \sum_{l=1}^{q} B_l H_{t-l} \tag{3.14}$$

where $H_t = (h_{1t},...,h_{mt})'$, $\omega = (\omega_1,...,\omega_m)'$, $D_t = diag(h_{i,t}^{1/2})$, $\eta_t = (\eta_{1t},...,\eta_{mt})'$, $\vec{\varepsilon}_t = (\varepsilon_{1t}^2,...,\varepsilon_{mt}^2)'$, A_k and B_l are $m \times m$ matrices with typical elements α_{ij} and β_{ij}, respectively, for i,j=1,…,m, and F_t is the past information available to time t. Spillover effects are given in the conditional volatility for each asset in the portfolio, specifically where A_k and B_l are not diagonal matrices. For the VARMA-GARCH model, the matrix of conditional correlations is given by $E(\eta_t \eta_t') = \Gamma$.

VARMA-AGARCH

An extension of the VARMA-GARCH model is the VARMA-AGARCH model of McAleer, M., et al. (2009), which assumes asymmetric impacts of positive and negative shocks of equal magnitude, and is given by:

$$H_t = \omega + \sum_{k=1}^{p} A_k \vec{\varepsilon}_{t-k} + \sum_{k=1}^{p} C_k I_{t-k} \vec{\varepsilon}_{t-k} + \sum_{l=1}^{q} B_l H_{t-l} \tag{3.15}$$

where C_k are $m \times m$ matrices for k = 1,…,p and I(η_t)=diag(I(η_{it})) is an $m \times m$ matrix, so that $I = \begin{cases} 0, \varepsilon_{k,t} > 0 \\ 1, \varepsilon_{k,t} \le 0 \end{cases}$. VARMA-AGARCH reduces to VARMA-GARCH when C_k =0 for all k.

CCC

If the model given by equation (3.15) is restricted so that C_k = 0 for all k, with A_k and B_l being diagonal matrices for all k,l, then VARMA-AGARCH reduces to:

$$h_{it} = \omega_i + \sum_{k=1}^{p} \alpha_i \varepsilon_{i,t-k} + \sum_{l=1}^{q} \beta_i h_{i,t-l} \tag{3.16}$$

Which is the constant conditional correlation (CCC) model of Bolerslev, T. (1990), for which the matrix of conditional correlations is given by $E(\eta_t \eta_t') = \Gamma$. As given in equation (3.16), the CCC model does not have volatility spillover effects across different financial assets, and does not allow conditional correlation coefficients of the returns to vary over time.

DCC

Engle, R.F. (2002) proposed the Dynamic Conditional Correlation (DCC) model. The DCC model can be written as follows:

$$y_t \mid F_{t-1} \sim (0, Q_t), \quad t = 1, \ldots, T \tag{3.17}$$

$$Q_t = D_t \Gamma_t D_t, \tag{3.18}$$

where $D_t = diag(h_{1t}^{1/2}, \ldots, h_{mt}^{1/2})$ is a diagonal matrix of conditional variances, with m asset returns, and F_t is the information set available at time t. The conditional variance is assumed to follow a univariate GARCH model, as follows:

$$h_{it} = \omega_i + \sum_{k=1}^{p} \alpha_{i,k} \varepsilon_{i,t-k} + \sum_{l=1}^{q} \beta_{i,l} h_{i,t-l} \tag{3.19}$$

When the univarate volatility models have been estimated, the standardized residuals, $\eta_{it} = y_{it} / \sqrt{h_{it}}$, are used to estimate the dynamic conditional correlations, as follows:

$$Q_t = (1 - \phi_1 - \phi_2)S + \phi_1 \eta_{t-1} \eta'_{t-1} + \phi_2 Q_{t-1} \quad (3.20)$$

$$\Gamma_t = \left\{ (diag(Q_t)^{-1/2} \right\} Q_t \left\{ (diag(Q_t)^{-1/2} \right\} \quad (3.21)$$

where S is the unconditional correlation matrix of the returns shocks, and equation (3.21) is used to standardize the matrix estimated in (3.20) to satisfy the definition of a correlation matrix. For details regarding the regularity conditional and statistical properties of DCC and the more general GARCC model, see McAleer, M., et at. (2008).

3.3 Volatility Index

This paper uses the Chicago Board Options Exchange (CBOE) volatility index (VIX) to represent the volatility index for the USA, and uses The Dow Jones EURO STOXX 50 volatility index (VSTOXX) to represent the volatility index for Europe. It provides a key measure of market expectations of near-term volatility based on the Dow Jones EURO STOXX 50 options prices. The Dow Jones EURO STOXX 50 index is a Blue-chip representation of sector leaders in the Euro zone. The index covers Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, The Netherlands, Portugal, and Spain.

The method to calculate volatility index follows:

Step 1: Calculate σ_1^2 and σ_2^2 (1= the near term options, 2 = the next term options*)

$$\sigma_i^2 = \frac{2}{T}\sum_i \frac{\Delta K_i}{K_i^2} e^{RT} Q(K_i) - \frac{1}{T}\left[\frac{F}{K_0} - 1\right]^2$$

Where σ is VIX/100 VIX = σ x 100

T Time to expiration

F Forward index level derived from index option prices

(Note: F = Strike price+ e^{RT} x (Call price – Put price)

K_i Strike price of ith out-of-the-money options; a call if $K_i > F$ and a put if $K_i < F$

ΔK_i Interval between strike prices-half the distance between the strike on either side of K_i: $\Delta K_i = \left(K_{i+1} - K_{i-1}\right)/2$

(Note: ΔK for the lowest strike is simply the difference between the lowest strike and the next higher strike. Likewise, ΔK for the highest strike is the difference between the highest strike and the next lower strike.)

K_0 First strike below the forward index level, F

R Risk-free interest rate to expiration

$Q(K_i)$ The midpoint of the bid-ask spread for each option with strike K_i.

Step 2: Interpolate σ_1^2 and σ_2^2 to arrive at a single value with a constant maturity of 30 days to expiration. Then take the square root of that value.

* The new VIX generally uses put and call options in the two nearest-term expiration months in order to bracket a 30-day calendar period. However, with 8 days left to expiration, the new VIX "rolls" to the second and third contract months in order to minimize pricing anomalies that might occur close to expiration.

$$\sigma = \sqrt{\left\{ T_1\sigma_1^2 \left[\frac{N_{T_2} - N_{30}}{N_{T_2} - N_{T_1}} \right] + T_2\sigma_2^2 \left[\frac{N_{30} - N_{T_1}}{N_{T_2} - N_{T_1}} \right] \right\} \times \frac{N_{365}}{N_{30}}}$$

Where N_{T_1} = Number of minutes to expiration of the near term options

N_{T_2} = Number of minutes to expiration of the next term options

N_{30} = Number of minutes in 30 days

N_{365} = Number of minutes in a 365-day year

Step 3: multiply by 100 to get VIX.

$$VIX = 100 \times \sigma$$

3.4 Data and Estimation

3.4.1 Data

The data used in the paper is the daily closing price sector indices of the S&P 500 and STOXX for the USA and Europe, respectively. The price sector indices of the S&P 500 and STOXX have 10 sectors, as shown in Table 3.1. However, this paper aggregates the price sector index by grouping sectors 1, 2, and 3 together, grouping sectors 4, 5, and 6 together, and grouping sectors 7, 8, 9, and 10 together. All the data is obtained from DataStream. The sample ranges from 23 January 1995 through 6 November 2008 with 3,476 observations for the USA, and 1 January 1992 through 6 November 2008 with 4,333 observations for Europe.

Two characteristics of the data, namely normality and stationarity, will be investigated before estimate univariate and multivariate analyses. Normality is an important issue in estimation since it is typically assumed in the maximum likelihood estimation (MLE) method; otherwise, the quasi-MLE (QMLE) method should be used. Stationarity is an important characteristic for time series data. If data is nonstationary, it will be necessary to difference the data before estimation because if not, the result will be spurious regression.

The normality of the variables can be seen from the Jarque-Bera (J-B) Lagrange multiplier test statistics in Table 3.2. As the probability associated with the J-B statistics is zero, it can be seen that the returns data is not normally distributed.

For the stationarity of data, this paper uses the Augmented Dicky Fuller (ADF) test. The test is given as follows:

$$\Delta y_t = \theta y_{t-1} + \sum_{i=1}^{p} \phi_i \Delta y_{t-i} + \varepsilon_t \tag{3.22}$$

$$\Delta y_t = \alpha + \theta y_{t-1} + \sum_{i=1}^{p} \phi_i \Delta y_{t-i} + \varepsilon_t \tag{3.23}$$

$$\Delta y_t = \alpha + \beta t + \theta y_{t-1} + \sum_{i=1}^{p} \phi_i \Delta y_{t-i} + \varepsilon_t \tag{3.24}$$

where equation (3.22) has no intercept and trend, equation (3.23) has intercept but no trend, and equation (3.24) has intercept and trend. The null hypothesis in equations (3.22), (3.23) and (3.24) are $\theta = 0$, which means that y_t is nonstationary (Dickey and Fuller, 1979). However, the ADF test accommodates serial correlation by explicitly modeling the structure of serial correlation, but not heteroscedasticity, while the Phillips-Perron (PP) tests accommodates both serial correlation and heteroscedasticity using non-parametric techniques. The PP test has also been shown to have higher power in finite samples than the ADF test (Phillips and Perron, 1988).

The PP test estimates as follows:

$$\Delta y_t = \theta y_{t-1} + x_t'\delta + \varepsilon_t \tag{3.25}$$

the test is evaluated using a modified t-ratio of the form:

$$\hat{t}_\alpha = t_\alpha \left(\frac{\gamma_0}{f_0}\right)^{1/2} - \frac{T\left(f_0 - \gamma_0\right)\left(se\left(\hat{\alpha}\right)\right)}{2 f_0^{1/2} s}$$

where $\hat{\alpha}$ is the estimate, t_α is the t-ratio of $\hat{\alpha}$, $se\left(\hat{\alpha}\right)$ is the standard error of $\hat{\alpha}$, and s is the standard error of the regression. In addition, γ_0 is a consistent estimate of the error variance in (3.25). The remaining f_0 is an estimator of the residual spectrum at frequency zero. The PP test is known as the non-augmented Dickey-Fuller test. The results of test stationary by using ADF test and PP test for the USA and Europe in Table 3.3 show that all the returns are stationary at the 1% level.

3.4.2 Estimation

The parameters in models (3.4), (3.5), (3.6), (3.7), (3.14), (3.15), (3.16), and (3.19) can be obtained by maximum likelihood estimation (MLE) using a joint normal density, as follows:

$$\hat{\theta} = \arg\min_{\theta} \frac{1}{2} \sum_{t=1}^{n} (\log|Q_t| + \varepsilon_t' Q_t^{-1} \varepsilon_t) \tag{3.26}$$

where θ denotes the vector of parameters to be estimated in the conditional log-likelihood function, and $|Q_t|$ denotes the determinant of Q_t, the conditional covariance matrix. When η_t does not follow a joint normal distribution, equation (3.26) is defined as the Quasi-MLE (QMLE).

3.5 Empirical Results

This paper use ARCH(1), GARCH(1,1), GJR(1,1), and EGARCH(1,1) models to estimate the single index model, and we assume that mean equation of all models have autoregressive terms (AR(1)). The results are shown in Table 3.4. The two entries for each parameter are the parameter estimate and Bollerslev-Wooldridge (1992) robust t-ratios. In the USA, mean equation is significant only in constant terms. Variance equation estimates are significant for all models except for ARCH effect in GJR model. For Europe, mean equation is significant in both constant terms and AR(1) terms, except the ARCH(1) model and all models in variance equation are significant. GJR dominates GARCH and ARCH. So, there is asymmetry, while EGARCH shows there is asymmetry but not leverage in Europe and the USA.

The portfolio model is estimated by using multivariate volatility as given in Tables 3.5 to 3.12. The multivariate volatility used in this paper is CCC, DCC, VARMA-GARCH, and VARMA-AGARCH. The results of VARMA-GARCH for the USA and Europe in Table 3.5 and 3.6, respectively, show negative effect of shock or news from RSP53FHI (RSP53CCE) sector to RSP53CCE (RSP53FHI) sector. In RSP53IMTU sector have negative effect of shock or news and positive effect of volatility spillover from RSP53FHI sector.

VARMA-GARCH for Europe is given in Table 3.6. The results show negative effect of shock or news from RSTFIIM sector to RSTABB sector and RSTCCF sector and negative effect of shock or news from RSTABB sector to RSTFIIM sector. Moreover, the results show volatility spillover from RSTFIIM sector to RSTABB sector and from RSTABB sector to RSTFIIM.

The results VARMA-AGARCH for the USA and Europe are given in Tables 3.7 and 3.8. Asymmetric effects are significant only for RSP53CCE sector for the USA and RSTABB sector for Europe.

In the USA, constant conditional correlations between the conditional volatilities of RSP53CCE sector and RSP53FHI sector for the CCC, VARMA-GARCH, and VARMA-AGARCH in Table 3.9 are identical at 0.789. Constant conditional correlations between the conditional volatilities of RSP53CCE sector and RSP53IMTU sector for the three models above are identical at 0.639. RSP53CCE sector and RSP53IMTU sector have constant conditional correlations between the conditional volatilities for the three models which are identical at 0.689.

Constant conditional correlations between the standardized residuals, of RSTABB sector and RSTCCF sector, RSTABB sector and RSTFIIM sector, and RSTCCF sector and RSTFIIM sector for the CCC, VARMA-GARCH, and VARMA-AGARCH in Table 3.10 are identical at 0.86, 0.88, and 0.84, respectively, in Europe.

From Tables 3.11 and 3.12, we can see that estimated coefficient is significant in both the USA and Europe market. Therefore the conditional correlations of the overall returns are dynamic.

3.6 Value-at-Risk

Value-at-Risk (VaR) needs to be provided to the appropriate regulatory authority at the beginning of the day, and is then compared with the actual returns at the end of the day. (see McAleer, M. (2008a))

For the purposes of the Basel II Accord penalty structure for violations arising from excessive risk taking, a violation is penalized according to its cumulative frequency of occurrence in 250 working days, which is shown in Table 3.13.

A violation occurs when VaR_t > negative returns at time t. Suppose that interest lies in modeling the random variable Y_t, which can be decomposed as follows (see McAleer, M. and da Veiga, B. (2008a):

$$Y_t = E(Y_t \mid F_{t-1}) + \varepsilon_t \tag{3.26}$$

This decomposition suggests that Y_t is comprised of a predictable component, $E(Y_t \mid F_{t-1})$, which is the conditional mean, and a random component, ε_t. The variability of Y_t, and hence its distribution, is determined entirely by the variability of ε_t. If it is assumed that ε_t follows a distribution such that:

$$\varepsilon_t \sim D(\mu_t, \sigma_t) \tag{3.27}$$

where μ_t and σ_t are the unconditional mean and standard deviation of ε_t, respectively, the VaR threshold for Y_t can be calculated as:

$$VaR_t = E(Y_t \mid F_{t-1}) - \alpha\sigma_t$$

where α is the critical value from the distribution of ε_t to obtain the appropriate confidence level. Alternatively, σ_t can be replaced by alternative estimates of the conditional variance to obtain an appropriate VaR. (see Section 3.2 and 3.3)

The Basel II encourages the optimization problem with the number of violations and forecasts of risk as endogenous choice variables, which are as follows:

$$\underset{\{k, VaR_t\}}{Minimize} \quad DCC_t = \max\left\{-(3+k)\overline{VaR}_{60}, -VaR_{t-1}\right\} \tag{3.28}$$

where DCC is daily capital charges, k is a violation penalty ($0 \le k \le 1$) (see Table 3.13), $\overline{VaR}_{60}$ is mean VaR over the previous 60 working days, and VAR_t is Value-at-Risk for day t.

This paper calculates VaR from the period of 4 January 1999 up to 6 November 2008 for Europe because VSTOXX has data starting from 4 January 1999. In the USA, we calculate VaR from the period of 24 January 1995 up to 6 November 2008. In order to simplify the analysis, we assumed that the portfolio returns are constant weights by using market capitalization at the first daily data. $E(Y_t \mid F_{t-1})$ is the expected returns for all models, and α is the critical value from the distribution of ε_t to obtain the appropriate confidence level of 1%.

Figures 3.1 – 3.4 show the VaR forecasts and realized returns of each single index models and portfolio models for the USA and Europe, respectively.

Table 3.14 shows the mean daily capital charge for the USA, in the single index models, ARCH(1) model has the highest at 12.353% and EGARCH(1,1) model has the lowest at 11.053% if compared with other ARCH-type models. However, the

Riskmetrics^{TM} model has the lowest at 10.855% in the single index models. ARCH(1) model has minimum number of violations at 21 times, and the lowest mean of absolute deviation of the violation from the VaR forecast at 1.736%. The Riskmetrics^{TM} model has maximum number of violations at 33 times, and the EGARCH(1,1) model has the highest mean of absolute deviation of the violation from the VaR forecast at 2.257%. GJR(1,1) has maximum number of violations at 25 times, which compares with the ARCH-type model.

In the portfolio models, DCC model has the lowest mean daily capital charge at 9.383%, the lowest the mean of absolute deviation of the violations from the VaR forecast at 1.563%, and the minimum number of violations at 16 times for all observations. VARMA-GARCH model has the highest mean daily capital charge at 9.599% and the highest the mean of absolute deviation of the violations from the VaR forecast at 1.867%. The CCC model has the maximum number of violations at 20 times for all observations. Table 3.14 also shows the model which uses VIX to calculate VaR has meant the daily capital charge is 10.091%, and the number of violations is 23 times for all observations.

The mean daily capital charge for Europe is shown in Table 3.15, in the single index models, ARCH(1) model has the highest at 15.437%, and minimum number of violations at 12 times. GJR(1,1) model has the lowest mean daily capital charge at 14.443% if compared with the ARCH-type model. However, the Riskmetrics^{TM} model has the lowest mean daily capital charge at 14.353% of all single index models. The EGARCH(1,1) model has the maximum number of violations at 23 times in ARCH-type model, and the highest mean of absolute deviation of the violation from the VaR forecast at 2.410%. However, the Riskmetrics^{TM} model has the maximum number of violations at 25 times.

In the portfolio models, the mean daily capital charge of the CCC model has the lowest at 11.991%. VARMA-GARCH model has the highest mean daily capital charge at 12.514%. The DCC model has the highest mean of absolute deviation of the violations from the VaR forecast at 1.968%, and the minimum number of violations at 18 times for all observations. The VARMA-AGARCH model has the highest mean of absolute deviation of the violation from the VaR forecast at 1.772%, and the maximum number of violations at 21 times for all observations. Table 3.15 also shows the model which uses VSTOXX to calculate VaR, and the mean daily capital charge is 13.714%.

The comparing in correlations of Value-at-Risk forecast calculated from various models for the USA is shown in Table 3.16 The results show that the correlations between the GARCH(1,1) model and the Riskmetrics^{TM} model are high because Riskmetrics^{TM} is subset of GARCH(1,1) model. The correlations between the GJR (1,1) model and the EGARCH(1,1) model also are highly correlated because the EGARCH(1,1) model shows there is asymmetry but not leverage. Moreover, the VARMA-GARCH model and the VARMA-AGARCH model are high correlation because the VARMA-AGARCH model show there is not asymmetry for all sectors.

The results of correlations of Value-at-Risk forecast for Europe is shown in Table 3.17. It shows that the correlations between the GJR (1,1) model and the EGARCH(1,1) model and the correlations between the VARMA-GARCH model and the VARMA-AGARCH model are highly correlated as same as the results of the USA. Moreover, the correlations between the CCC model and the DCC model in

Europe are also high. Therefore, the DCC model is better than the CCC model in term of the statistic, but the CCC model are better than the DCC model in term of practice.

3.7 Conclusion

Volatility forecasting is an important task in financial markets. In 1993, the Chicago Board Options Exchange (CBOE) introduced the CBOE volatility index, VIX, and it quickly became the benchmark for stock market volatility. In Europe there is also a volatility index, which is calculated by the same method of CBOE. The volatility index in Europe is the VSTOXX volatility index, which was introduced on 20 April 2005. However, the volatility index uses the model-free implied volatility, which is not based on a specific volatility model.

This paper would like to construct an index of volatility by using conditional volatility models by: (1) fitting a univariate volatility model to the portfolio returns (hereafter called the single index model (see McAleer and de Veiga (2008a,2008b)); or (2) using a multivariate volatility model to forecast the conditional variance of each asset in the portfolio, as well as the conditional correlations between all asset pairs, in order to calculate the forecasted portfolio variance (hereafter called the portfolio model) for the USA and Europe. Then the index of volatility is compared with the volatility index and Riskmetrics^{TM} by using the predictive power of Value-at-Risk.

The univariate volatility models used in this paper are ARCH(1), GARCH(1,1), GJR(1,1), and EGARCH(1,1) which means the equations have constant term and autoregressive term (AR(1)). For the multivariate volatility model, we used CCC, DCC, VARMA-GARCH, VARMA-AGARCH models, which means the equations have constant term and autoregressive term (AR(1)), the same as the univariate volatility model.

If we consider the mean daily capital charge, the results show that the Riskmetrics^{TM} model dominates the other models in the single index model for the USA and Europe. However, if we compare between ARCH-type, the EGARCH(1,1) model dominates the other models for the USA. However, the GJR(1,1) model dominates the other models in Europe. In portfolio model, the DCC model dominates the other models for the USA. Immediate CCC model dominates the other models for Europe. If we compare the mean daily capital charge of the index of volatility, which uses the single index model and volatility index (i.e. VIX and VSTOXX), the results show that the VIX and VSTOXX are dominate for the single index model. However, if we compare the index of volatility, which uses the portfolio models with volatility index (VIX or VSTOXX), the results show that the portfolio models dominate the volatility index because the portfolio models have a lower mean daily capital charge compared to the volatility index. The higher daily capital charge has an effect on the profitability of the financial institution.

Moreover, the correlations of Value-at-Risk forecast calculated from the GJR (1,1) model and the EGARCH(1,1) model are highly correlated for both the USA and Europe because they have asymmetric but not leverage. Moreover, the correlations of VaR forecast between the VARMA-GARCH model and the VARMA-AGARCH model are also high for both the USA and Europe because asymmetric effect in the VARMA-AGARCH model are significant only one sector in both the USA and Europe. Finally, the statistic of the DCC model for Europe is significant so that the DCC model is better than the CCC model. However, in practice the CCC model is better than the DCC model because it has lowest mean daily capital charge.

Table 3.1 Summary of Variable Names

The USA	**Price Sector Index Names**	**Variable Names**
	S&P500 CONSUMER DISCRETIONARY S&P500 CONSUMER STAPLES S&P500 ENERGY	RSP53CCE
	S&P500 FINANCIALS S&P500 HEALTH CARE S&P500 INDUSTRIALS	SP53FHI
	S&P500 INFORMATION TECHNOLOGY S&P500 MATERIALS S&P500 TELECOMMUNICATION SERVICES S&P500 UTILITIES	SP53IMTU
Europe	DJ EURO STOXX AUTOMOBILES & PARTS DJ EURO STOXX BANKS DJ EURO STOXX BASIC RESOURCES	STABB
	DJ EURO STOXX CHEMICALS DJ EURO STOXX CONSTRUCTION & MATERIALS DJ EURO STOXX FINANCIAL SERVICES	STCCF
	DJ EURO STOXX FOOD & BEVERAGE DJ EURO STOXX INDUSTRIAL GOODS & SERVICES DJ EURO STOXX INSURANCE DJ EURO STOXX MEDIA	STFIIM

Source: DataStream.

Table 3.2 Jarque-Bera Test of Normality and Probability for Returns

Country	Returns	Jarque-Bera	Probability
The USA	RPORTSP53	8789.624	0.000
	RSP53CCE	39039.39	0.000
	RSP53FHI	6186.943	0.000
	RSP53IMTU	3676.259	0.000
Europe	RPORTST3	12404.28	0.000
	RSTABB	227469.0	0.000
	RSTCCF	12175.59	0.000
	RSTFIIM	8260.200	0.000

Note: RPORTSP53 represent portfolio return for the USA

RPORTST3 represent portfolio return for the Europe

Table 3.3 Unit Root Test of Returns for the USA and Europe

Countries	Variables	Trend and intercept	Intercept	None
Augmented Dickey-Fuller test				
USA	RPORTSP53	**-45.410**	**-45.291**	**-45.267**
	RSP53CCE	**-21.347**	**-21.272**	**-21.200**
	RSP53FHI	**-44.292**	**-44.128**	**-44.110**
	RSP53IMTU	**-44.339**	**-44.279**	**-44.278**
Europe	RPORTST3	**-64.245**	**-64.212**	**-64.207**
	RSTABB	**-49.016**	**-48.981**	**-48.977**
	RSTCCF	**-63.379**	**-63.368**	**-63.348**
	RSTFIIM	**-62.602**	**-62.562**	**-62.561**
Phillips-Perron Test				
USA	RPORTSP53	**-62.651**	**-62.352**	**-62.286**
	RSP53CCE	**-63.513**	**-63.431**	**-63.242**
	RSP53FHI	**-60.364**	**-59.767**	**-59.705**
	RSP53IMTU	**-60.309**	**-60.208**	**-60.206**
Europe	RPORTST3	**-64.224**	**-64.193**	**-64.187**
	RSTABB	**-67.088**	**-67.066**	**-67.056**
	RSTCCF	**-63.380**	**-63.357**	**-63.316**
	RSTFIIM	**-62.524**	**-62.477**	**-62.477**

Note: Entries in bold are significant at the 99% level.

Table 3.4 Single Index Model for the USA and Europe

Variable	Model	Mean equation		Variance equation			
		C	AR(1)	ϖ	α	γ	β
RPORTSP53	ARCH(1)	**0.0414**	-0.137	**1.174**	**0.301**		
		2.263	-1.9156	**16.357**	**5.487**		
	GARCH(1,1)	**0.066**	-0.026	**0.008**	**0.070**		**0.928**
		4.572	-1.52	**2.433**	**6.121**		**89.919**
	GJR(1,1)	**0.035**	-0.012	**0.011**	-0.009	**0.125**	**0.938**
		2.400	-0.735	**4.452**	-0.875	**6.618**	**106.635**
	EGARCH(1,1)	**0.030**	-0.014	**-0.088**	**0.117**	**-0.103**	**0.982**
		2.021	0.017	**-6.257**	**6.505**	**-6.659**	**297.543**
RPORTST3	ARCH(1)	**0.049**	0.082	**1.017**	**0.419**		
		2.404	1.075	**15.582**	**7.826**		
	GARCH(1,1)	**0.053**	**0.045**	**0.015**	**0.098**		**0.894**
		4.113	**2.750**	**3.871**	**7.467**		**73.446**
	GJR(1,1)	**0.031**	**0.047**	**0.017**	**0.035**	**0.098**	**0.902**
		2.332	**2.960**	**4.730**	**2.507**	**5.026**	**84.428**
	EGARCH(1,1)	**0.0298**	**0.040**	**-0.130**	**0.170**	**-0.071**	**0.983**
		2.204	**2.450**	**-7.859**	**7.825**	**-5.023**	**282.238**

Notes: (1) The 2 entries for each parameter are the parameter estimate and Bollerslev-Wooldridge(1992) robust t-ratios.

(2) Entries in bold are significant at the 95% level

Table 3.5 Portfolio Models for the USA: VARMA-GARCH

Sectors	ϖ	α_{CCE}	β_{CCE}	α_{FHI}	β_{FHI}	α_{IMTU}	β_{IMTU}
RSP53CCE	0.005	**0.124**	**0.836**	**-0.062**	0.144	-0.014	-0.024
	1.753	**6.604**	**13.157**	**-2.185**	1.718	-0.816	-0.554
RSP53FHI	0.004	**-0.071**	0.141	**0.126**	**0.797**	-0.033	0.092
	1.062	**-4.437**	1.622	**4.662**	**9.540**	-1.938	1.694
RSP53IMTU	0.003	-0.006	-0.013	**-0.032**	**0.046**	**0.076**	**0.915**
	0.696	-1.006	-0.670	**-3.686**	**2.034**	**7.265**	**70.325**

Notes: (1) The 2 entries for each parameter are the parameter estimate and Bollerslev-Wooldridge(1992) robust t-ratios.

(2) Entries in bold are significant at the 95% level

Table 3.6 Portfolio Models for Europe: VARMA-GARCH

Sectors	ϖ	α_{ABB}	β_{ABB}	α_{CCF}	β_{CCF}	α_{FIIM}	β_{FIIM}
RSTABB	-0.005	**0.218**	0.332	-0.051	0.129	**-0.113**	**0.425**
	-0.437	**5.297**	1.955	-1.119	0.852	**-5.500**	**5.975**
RSTCCF	**0.028**	-0.047	0.162	**0.128**	**0.597**	**-0.057**	0.051
	3.589	-1.369	0.836	**3.161**	**5.860**	**-2.235**	0.421
RSTFIIM	0.006	**-0.134**	**0.559**	-0.020	**0.215**	**0.212**	**0.519**
	0.704	**-4.140**	**3.986**	-0.652	**2.047**	**8.871**	**3.478**

Notes: (1) The 2 entries for each parameter are the parameter estimate and Bollerslev-Wooldridge(1992) robust t-ratios.

(2) Entries in bold are significant at the 95% level

Table 3.7 Portfolio Models for the USA: VARMA-AGARCH

Sectors	ϖ	α_{CCE}	β_{CCE}	α_{FHI}	β_{FHI}	α_{IMTU}	β_{IMTU}	γ
RSP53CCE	0.006	**0.076**	**0.847**	**-0.068**	0.106	-0.006	-0.054	**0.061**
	1.907	**4.494**	**16.810**	**-3.245**	1.824	-0.367	-1.133	**5.030**
RSP53FHI	**0.007**	**-0.067**	**0.144**	**0.089**	**0.840**	**-0.048**	**0.132**	-4.811
	2.238	**-5.087**	**2.092**	**4.563**	**13.920**	**-2.802**	**2.291**	0.000
RSP53IMTU	0.007	-0.006	-0.0156	**-0.030**	**0.036**	**0.049**	**0.910**	0.027
	1.726	-1.109	-1.010	**-5.273**	**2.073**	**4.894**	**67.129**	0.000

Notes: (1) The 2 entries for each parameter are the parameter estimate and Bollerslev-Wooldridge(1992) robust t-ratios.

(2) Entries in bold are significant at the 95% level

Table 3.8 Portfolio Models for Europe: VARMA-AGARCH

Sectors	ϖ	α_{ABB}	β_{ABB}	α_{CCF}	β_{CCF}	α_{FIIM}	β_{FIIM}	γ
RSTABB	-0.006	**0.192**	0.310	-0.058	0.156	**-0.115**	0.462	**0.051**
	-0.255	**3.77**	0.762	-1.067	0.541	**-2.493**	1.295	**3.191**
RSTCCF	**0.029**	-0.042	0.191	**0.118**	**0.578**	-0.072	0.119	-0.006
	2.014	-1.404	0.701	**3.905**	**4.801**	-1.330	0.330	0.000
RSTFIIM	0.004	**-0.149**	0.568	-0.038	0.221	**0.209**	**0.436**	-6.912
	0.160	**-3.264**	1.928	-0.576	0.984	**6.621**	**5.208**	0.000

Notes: (1) The 2 entries for each parameter are the parameter estimate and Bollerslev-Wooldridge(1992) robust t-ratios.

(2) Entries in bold are significant at the 95% level

Table 3.9 Constant Conditional Correlations between Sectors Returns for the USA

Model	$\rho_{CCE, FHI}$	$\rho_{CCE, IMTU}$	$\rho_{FHI, IMTU}$
CCC	**0.764**	**0.623**	**0.678**
	118.173	**66.102**	**76.010**
VARMA-GARCH(1,1)	**0.789**	**0.639**	**0.689**
	95.676	**54.975**	**69.040**
VARMA-GARCH(1,1)	**0.789**	**0.639**	**0.689**
	95.677	**54.975**	**69.040**

Notes: (1) The 2 entries for each parameter are the parameter estimate and Bollerslev-Wooldridge(1992) robust t-ratios.

(2) Entries in bold are significant at the 95% level

Table 3.10 Constant Conditional Correlations between Sectors Returns for Europe

Model	$\rho_{ABB, CCF}$	$\rho_{ABB, CCF}$	$\rho_{CCF, FIIM}$
CCC	**0.848**	**0.865**	**0.831**
	146.682	**111.551**	**114.528**
VARMA-GARCH(1,1)	**0.862**	**0.880**	**0.848**
	142.523	**179.222**	**155.775**
VARMA-GARCH(1,1)	**0.860**	**0.879**	**0.846**
	122.988	**127.783**	**117.031**

Notes: (1) The 2 entries for each parameter are the parameter estimate and Bollerslev-Wooldridge(1992) robust t-ratios.

(2) Entries in bold are significant at the 95% level

Table 3.11 DCC-GARCH(1,1) Estimates for the USA

Model	ϕ_1	ϕ_2
$Q_t = (1-\phi_1-\phi_2)S + \phi_1\eta_{t-1}\eta'_{t-1} + \phi_2 Q_{t-1}$	**0.034** **10.070**	**0.963** **245.762**

Notes: (1) The 2 entries for each parameter are the parameter estimate and Bollerslev-Wooldridge(1992) robust t-ratios.

(2) Entries in bold are significant at the 95% level

Table 3.12 DCC-GARCH(1,1) Estimates for Europe

Model	ϕ_1	ϕ_2
$Q_t = (1-\phi_1-\phi_2)S + \phi_1\eta_{t-1}\eta'_{t-1} + \phi_2 Q_{t-1}$	**0.030** **5.758**	**0.970** **182.163**

Notes: (1) The 2 entries for each parameter are the parameter estimate and Bollerslev-Wooldridge(1992) robust t-ratios.

(2) Entries in bold are significant at the 95% level

Table 3.13 Basel Accord Penalty Zones

Zone	Number of Violations	Increase in k
Green	0 to 4	0.00
Yellow	5	0.40
	6	0.50
	7	0.65
	8	0.75
	9	0.85
Red	10+	1.00

Note: The number of violations is given for 250 business days.

Table 3.14 Mean Daily Capital Charge and AD of Violations for the USA

Model	Number of Violations		Mean Daily Capital Charge	AD of Violations	
	All observation	250 trading day		Maximum	Mean
ARCH	21	2	12.353	3.706	1.736
GARCH	24	2	11.234	4.448	1.938
GJR	25	2	11.084	4.590	2.245
EGARCH	24	2	11.053	4.886	2.257
Riskmetrics^{TM}	33	2	10.855	1.827	1.827
CCC	20	1	9.471	4.972	1.749
DCC	16	1	9.383	5.199	1.563
VARMA-GARCH	17	1	9.599	5.085	1.867
VARMA-AGARCH	17	1	9.515	5.326	1.646
VIX	23	2	10.091	3.319	3.319

Note: (1) Number of Violations are a greater number of violations than would reasonably be expected given the specified confidence level of 1%.

(2) AD is the absolute deviation of the violations from the VaR forecast.

Table 3.15 Mean Daily Capital Charge and AD of Violations for Europe

Model	Number of Violations		Mean Daily Capital Charge	AD of Violations	
	All observation	250 trading day		Maximum	Mean
ARCH	12	1	15.437	2.148	1.001
GARCH	22	2	14.510	2.860	1.906
GJR	22	2	14.443	2.860	1.906
EGARCH	23	2	14.519	2.410	2.410
Riskmetrics^{TM}	25	2	14.353	0.000	0.000
CCC	20	2	11.991	2.918	1.910
DCC	18	2	12.437	3.759	1.968
VARMA-GARCH	19	2	12.514	3.348	1.885
VARMA-AGARCH	21	2	12.438	2.517	1.772
VSTOXX	21	2	13.714	1.155	0.774

Note: (1) Number of Violations are a greater number of violations than would reasonably be expected given the specified confidence level of 1%.

(2) AD is the absolute deviation of the violations from the VaR forecast.

Table 3.16 Correlations of Value-at-Risk forecasts for the USA

	ARCH	GARCH	GJR	EGARCH	RISKMETRICS^{TM}	CCC	DCC	VARMA-GARCH	VARMA-AGARCH	VIX
ARCH	1	0.555	0.569	0.571	0.528	0.518	0.536	0.479	0.489	0.456
GARCH	0.555	1	0.963	0.949	**0.992**	0.981	**0.991**	0.925	0.911	0.871
GJR	0.569	0.963	1	**0.984**	0.953	0.940	0.953	0.888	0.905	0.868
EGARCH	**0.571**	0.949	**0.984**	1	0.942	0.930	0.937	0.884	0.902	0.875
RISKMETRICS^{TM}	0.528	**0.992**	0.953	0.942	1	0.978	0.985	0.926	0.912	**0.885**
CCC	0.518	0.981	0.940	0.930	0.978	1	0.991	0.972	0.957	0.859
DCC	0.536	0.991	0.953	0.937	0.985	**0.991**	1	0.937	0.921	0.872
VARMA-GARCH	0.479	0.925	0.888	0.884	0.926	0.972	0.937	1	**0.992**	0.798
VARMA-AGARCH	0.489	0.911	0.905	0.902	0.912	0.957	0.921	**0.992**	1	0.800
VIX	0.456	0.871	0.868	0.875	0.885	0.859	0.872	0.798	0.800	1

Note: Entries in bold are highest correlation

Table 3.17 Correlations of Value-at-Risk forecasts for the Europe

	ARCH	GARCH	GJR	EGARCH	RISKMETRICS™	CCC	DCC	VARMA_GARCH	VARMA_AGARCH	VSTOXX
ARCH	1	0.613	0.617	0.615	0.550	0.565	0.557	0.473	0.471	0.485
GARCH	0.613	1	0.986	0.978	0.974	0.982	0.985	0.844	0.834	0.835
GJR	**0.617**	**0.986**	1	**0.991**	0.958	0.961	0.964	0.820	0.819	0.842
EGARCH	0.615	0.978	**0.991**	1	0.961	0.952	0.960	0.804	0.804	**0.867**
RISKMETRICS™	0.550	0.974	0.958	0.961	1	0.974	0.985	0.841	0.832	0.862
CCC	0.565	0.982	0.961	0.952	0.974	1	**0.993**	0.920	0.910	0.839
DCC	0.557	0.985	0.964	0.960	**0.985**	**0.993**	1	0.875	0.864	0.855
VARMA_GARCH	0.473	0.844	0.820	0.804	0.841	0.920	0.875	1	**0.998**	0.746
VARMA_AGARCH	0.471	0.834	0.819	0.804	0.832	0.910	0.864	**0.998**	1	0.746
VSTOXX	0.485	0.835	0.842	0.867	0.862	0.839	0.855	0.746	0.746	1

Note: Entries in bold are highest correlation

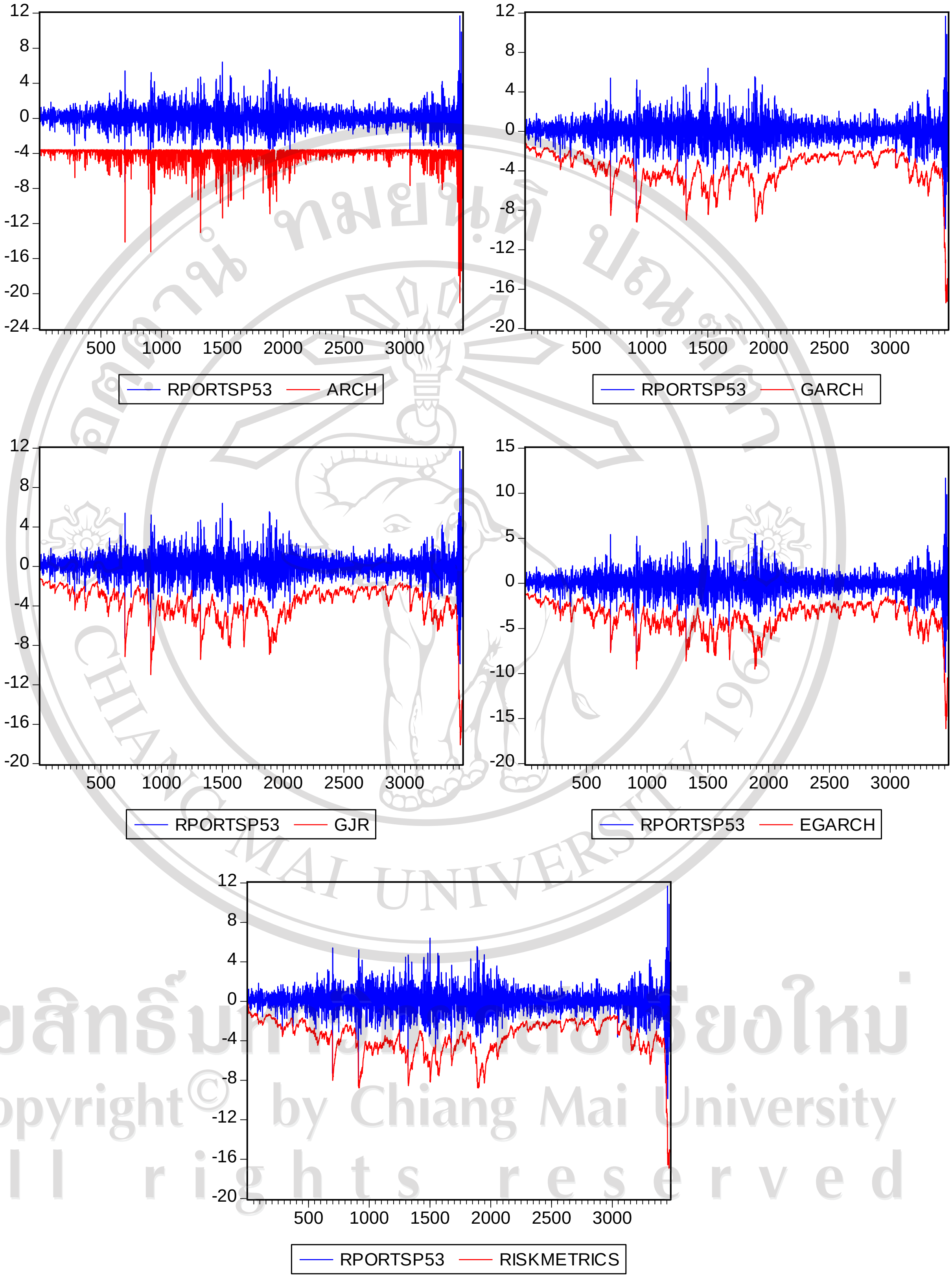

Figure 3.1 Single Index Models and Realized Returns VaR Forecasts for the USA

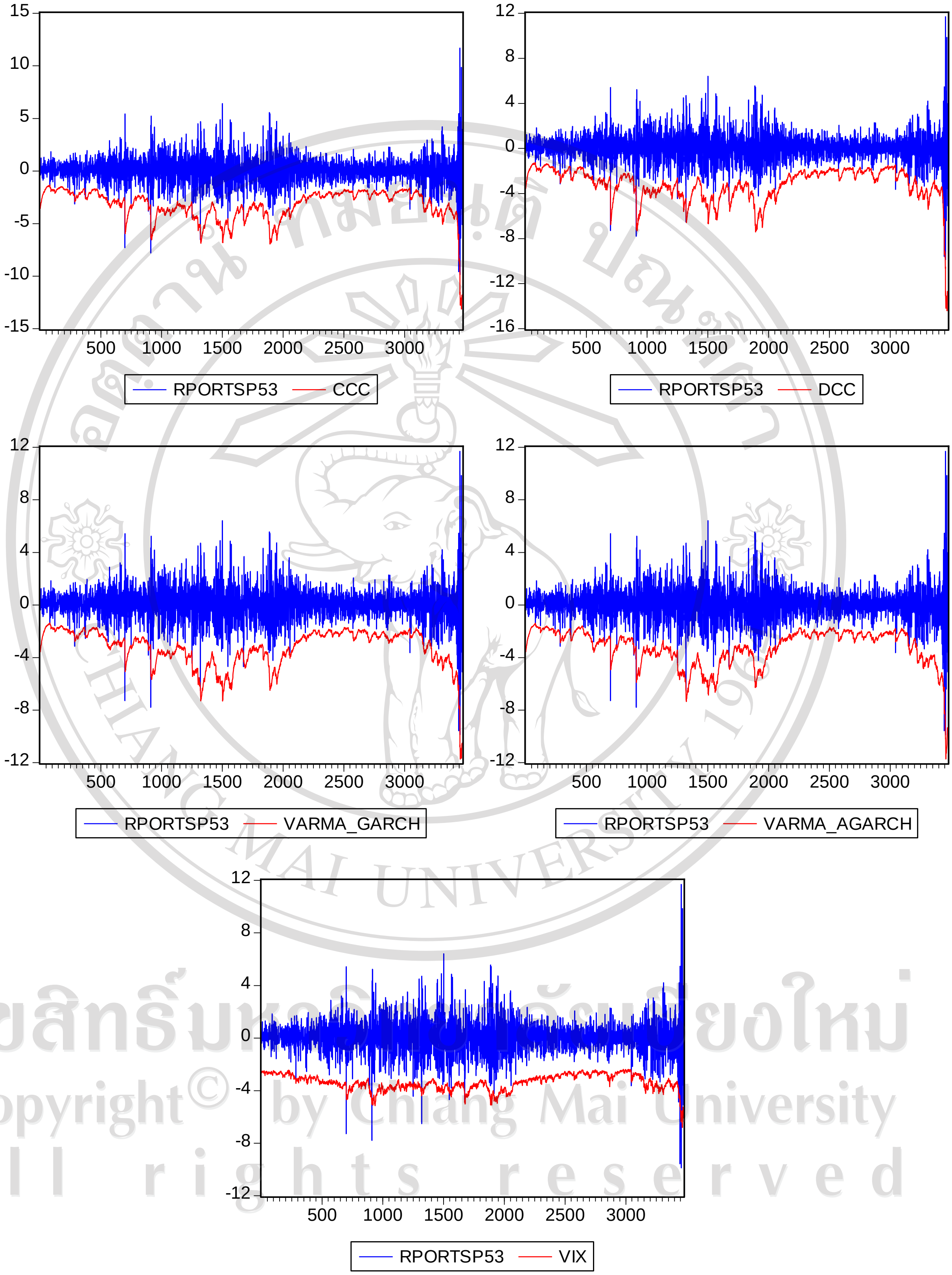

Figure 3.2 Portfolio Models, VIX and Realized Returns VaR Forecasts for the USA

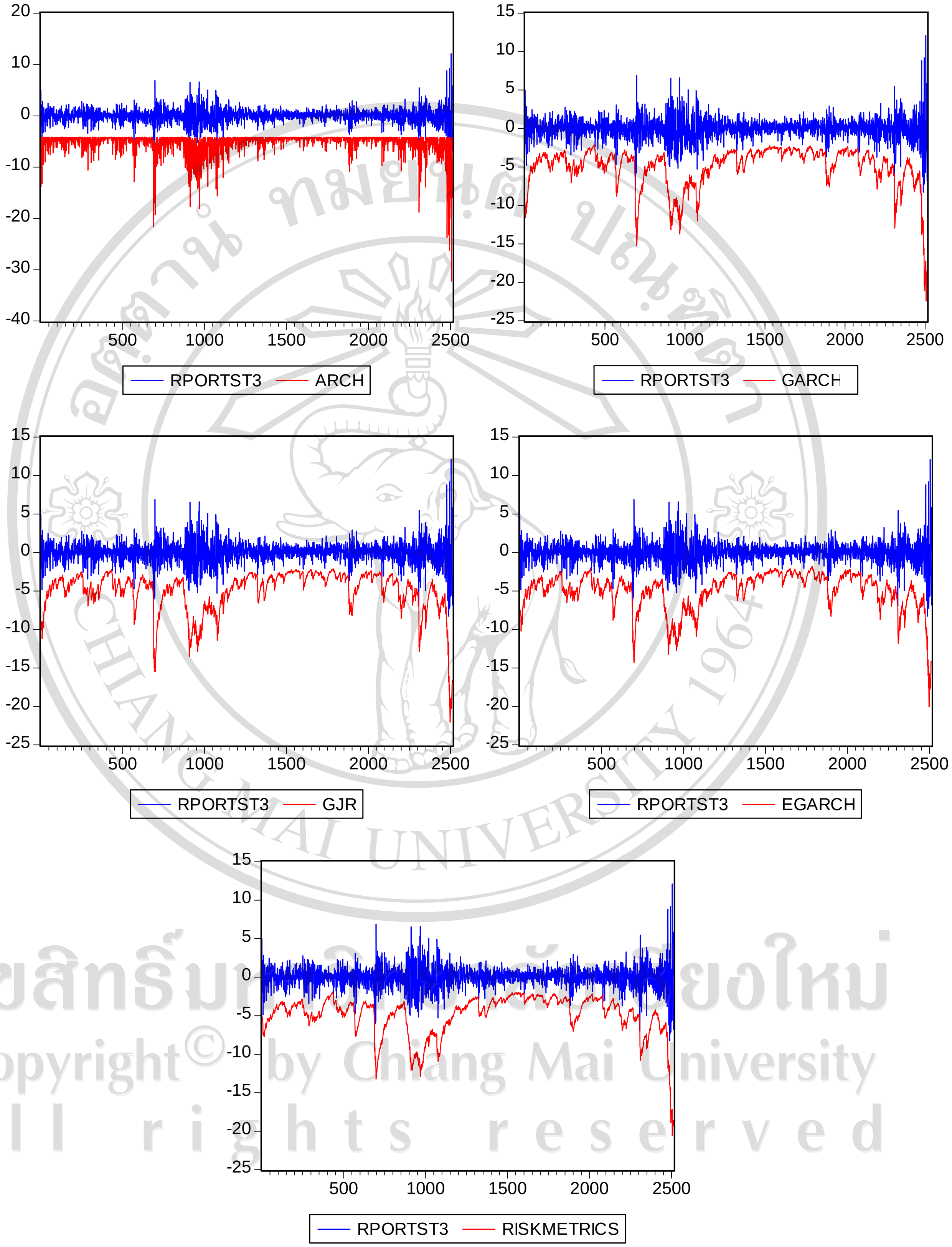

Figure 3.3 Single Index Models and Realized Returns VaR Forecasts for Europe

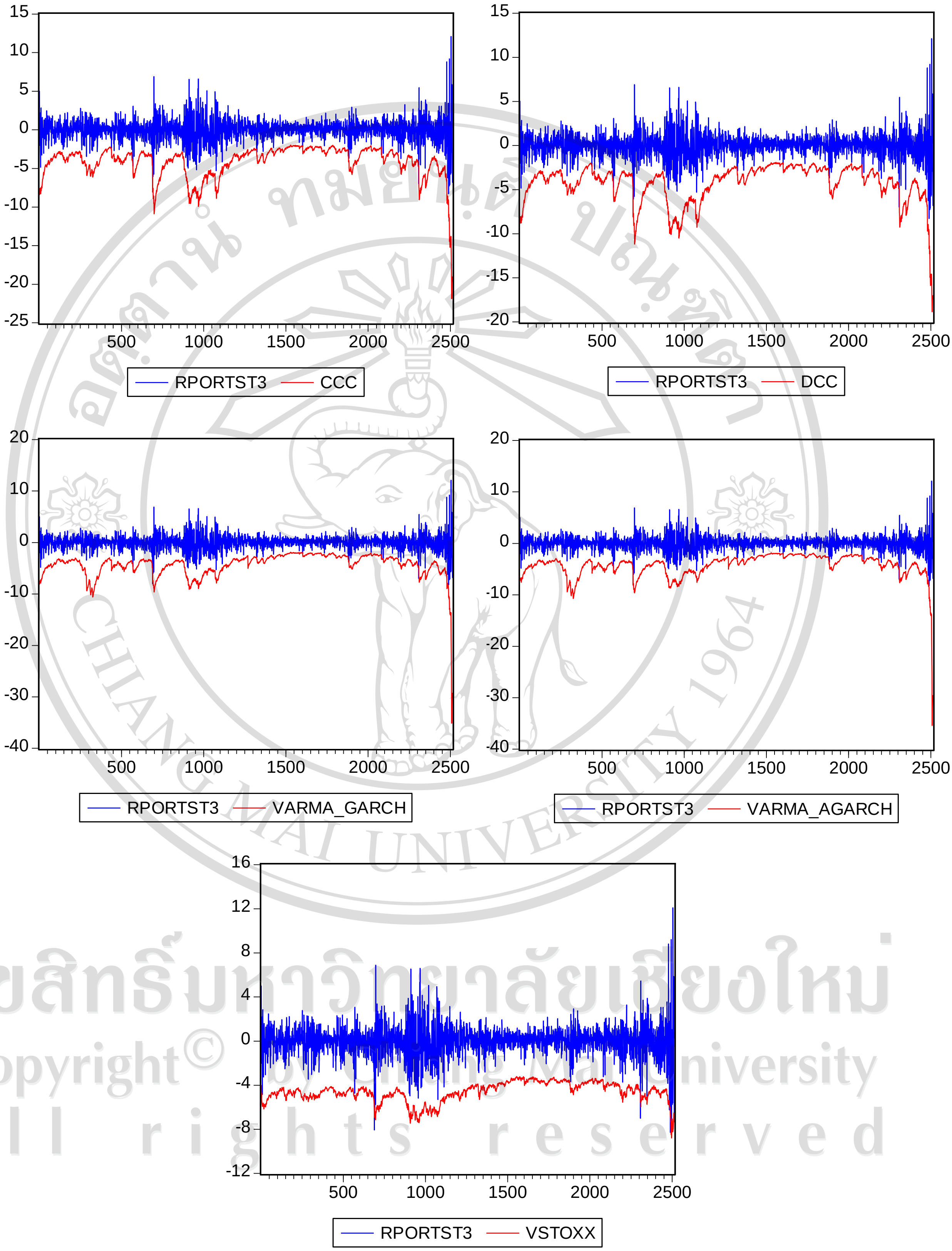

Figure 3.4 Portfolio Models, VSTOXX and Realized Returns VaR Forecasts for Europe