APPENDICES

APPENDIX A

A Simple Expected Volatility (SEV) Index: Application to SET50 Index Options

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A Simple Expected Volatility (SEV) Index: Application to SET50 Index Options*

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ABSTRACT

In 2003, the Chicago Board Options Exchange (CBOE) made two key enhancements to the volatility index (VIX) methodology based on S&P options. The new VIX methodology seems to be based on a complicated formula to calculate expected volatility. In this paper, with the use of Thailand's SET50 Index Options data, we modify the apparently complicated VIX formula to a simple relationship, which has a higher negative correlation between the VIX for Thailand (TVIX) and SET50 Index Options. We show that TVIX provides more accurate forecasts of option prices than the simple expected volatility (SEV) index, but the SEV index outperforms TVIX in forecasting expected volatility. Therefore, the SEV index would seem to be a superior tool as a hedging diversification tool because of the high negative correlation with the volatility index.

1. Introduction

Volatility is the key in portfolio and risk management especially in modern financial theory. It becomes the important tool for fund managers or investors to make decision in investment. Fund managers and investors tend to move the funds from the market that has high volatility to the market that has low volatility, for example, they can move funds from one stock market to other stock markets if the volatility in that stock market is increased.

In 1993, the Chicago Board Options Exchange (CBOE) introduced the CBOE Volatility Index, VIX, which quickly became the benchmark for stock market volatility. As volatility often signifies financial turmoil, the index is often referred to as the "investor fear gauge". The index is based on real-time option prices, and reflects investors' consensus view of future expected stock market volatility.

In September 2008, options trading become an even more important profit tool than a risk diversification tool from investors. The U.S. SEC, U.K. FSA., and Australia stepped into stop short-selling for financial companies in

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order to stabilize those companies. Recently options have become a significant diversification tool for investors to hedge their portfolios in both expected uptrend and (especially) downturn markets.

The trading volume in SPX options set a new record as 2,182,562 contracts were traded on 6 October 2008, with an average volume of 670,629 contracts per day. On 18 September, the total options volume exceeded 30 million contracts for the first time in history, from the previous day's record of 26 million contracts. Moreover, in the hamburger crisis, the Thailand SET50 options volume increased by 33.5% and 33% in September and October, respectively, as compared with August 2008.

One of the keys to options trading is leveraging, whereby leverage allows traders to make a significant amount of money from a relatively small change in price. The trader enjoys the ability of less money at a low investment for bigger bets to hedge a portfolio. In addition, the options trader can minimize exposure to risk from stock investment as a hedge of an under-priced asset relative to its fair value.

In 29 October 2007, the Stock Exchange of Thailand (SET), with the sub-company Thailand Futures Exchange (TFEX), launched the European-style options written on TFEX with ticker S50myycall/put strike price. For example, S50H09C600 denotes SET50 contract month of March in the year 2009 call option at the strike of 600. The contract multipliers of the options contracts are 200 Baht per index point.

In a competitive market, Singapore and Thailand are planning to integrate the Asian stock market to be more competitive to the world. TFEX should introduce innovative new products to attract foreign investors to invest and hedge their portfolios in Thailand.

The primary purpose of this paper is to simplify the apparently complicated expected volatility formula into a simpler relationship, with the use of SET50 index data becoming a simple expected volatility (SEV) index, and to adapt the new VIX calculation from CBOE to derive an implied volatility index (TVIX) for Thailand SET50 index options. Then we substitute the expected volatilities into the Black-Scholes model to predict call and put option prices.

The remainder of the paper is as follows. The volatility index is discussed in Section 2, a brief overview of the volatility index (VIX) from CBOE is given in Section 3, the new VIX formula is presented in Section 4, followed by a simple expected volatility index (SEV) in Section 5, the SET50 index options data for empirical analysis are discussed in Section 6, the Black-Scholes model for substituting the expected volatility to predict call and put option prices is discussed in Section 7, estimation is given in Section 8, and some concluding remarks are presented in Section 9.

2. Volatility Index

The idea of estimating implied volatility from options is relatively simple. There is no straightforward method to extract the information. With the large number of option pricing models, many researchers have applied various methods of estimating implied volatilities from option pricing models, especially the Black-Scholes model (see Black and Scholes (1973)). The model was originally developed to estimate implied volatility at each exercise price, as in Melino and Turnbull (1990), Nandi (1996), and Bakshi, Cao and Chen (1997).

Option prices calculate implied volatility that represents a market-based estimate of future price volatility, so that implied volatility is regarded as a fear gauge (Whaley (2000)). Implied volatilities are reported by investors, financial news services and other finance professionals. The information content and forecast quality of implied volatility is an important topic in financial markets research.

Latane and Rendleman (1976), Chiras and Manaster (1978), Beckers (1981) and Jorion (1995) provided early assessments of the forecast quality of implied volatility. They concluded that implied volatilities outperform historical standard deviations, although perhaps biased, as a good predictor of future volatility. Christensen and Prabhala (1998) found that implied volatility forecasts are biased, but dominate historical volatility in terms of ex ante forecasting power. Fleming (1998) used a similar volatility measure to show that implied volatilities outperform historical information.

Fleming *et al.* (1995) showed that implied volatilities from S&P100 index options yield efficient forecasts of one-month ahead S&P100 index return volatility, and can also eliminate mis-specification problems. Blair *et al.* (2001) concluded that the VIX index provides the most accurate forecasts for low- or high-frequency observations, and are also unbiased.

Dennis *et al.* (2006) found that daily innovations in VIX contain very reliable incremental information about the future volatility of the S&P100 index. Other studies that attempt to forecast implied volatility or use the information contained in implied volatility to trade in option markets include Harvey and Whaley (1992), Noh *et al.* (1994), and Poon and Pope (2000).

3. VIX from CBOE

VIX measures market expectation of near term volatility conveyed by stock index option prices. The original VIX was constructed using the implied volatilities of eight different S&P100 (OEX) option series so that, at any given time, it represented the implied volatility of an hypothetical at-the-money OEX option with exactly 30 days to expiration from an option-pricing model.

In 2003, the CBOE made two key enhancements to the VIX methodology. The new VIX is based on an up-to-the-minute market estimation of expected volatility that is calculated by using real-time S&P500 Index (SPX) option bid/ask quotes, and incorporates information from the volatility "skew" by using a wider range of strike prices rather than just at-the-money series with the market's expectation of 30-day volatility, and using nearby and second nearby options.

Until 2006, VIX was trading on the CBOE. The VIX options contract is the first product on market volatility to be listed on an SEC-regulated securities exchange. This new product can be traded from an options-approved securities account. Many investors consider the VIX Index to be the world's premier barometer of investor sentiment and market volatility, and VIX options are a very powerful risk management tool. VIX is quoted in percentage points, just like the standard deviation of a rate of return.

4. New VIX Procedure

The New VIX is more robust because it pools the information from option prices over the whole volatility skew, and not just from at-the-money options. The formula used in the new VIX calculation is given by the CBOE as follows:

$$\sigma^2 = \frac{2}{T}\sum_i \frac{\Delta K_i}{K_i^2} e^{RT} Q(K_i) - \frac{1}{T}\left[\frac{F}{K_0} - 1\right]^2$$

where

σ = VIX / 100 (so that VIX = σ x 100),

T = Time to expiration (in minutes),

F = Forward index level, derived from index option prices (based on at-the-money option prices, the difference between call and put prices is smallest).

The formula used to calculate the forward index level is:

F = Strike price (at-the-money) + e^{RT} x (Call price – Put price),

where

R = risk-free interest rate is assumed to be 3.01% (for simplicity, the government T-bills 3 month contract interest rate is used, as the Thailand options contract is a 3 months contract);

T = $\{M_{current\ day} + M_{settlement\ day} + M_{other\ days}\}$/minutes in a year,

where

$M_{current\ day}$ = # of minutes remaining until midnight of the current day,

$M_{settlement\ day}$ = # of minutes from midnight until 9:45 am on the TFEX settlement day,

$M_{other\ days}$ = Total # of minutes in the days between the current day and the settlement day;

K_i = Strike price of i^{th} out-of-the-money option; a call if K_i > F and a put if K_i < F;

ΔK_i = Interval between strike prices - half the distance between the strike on either side of K_i: $\Delta K_i = \frac{K_{i+1} - K_{i-1}}{2}$.

K_0 = First strike below the forward index level, F;

$Q(K_i)$ = The midpoint of the bid-ask spread for each option with strike K_i.

(Note: ΔK_i for the lowest strike is simply the difference between the lowest strike and the next higher strike. Likewise, ΔK for the highest strike is the difference between the highest strike and the next lower strike.)

With the adaptation of the VIX calculation to Thailand SET 50 index options, the Thailand expected volatility (TVIX) can be estimated.

5. A Simple Expected Volatility Index (SEV Index)

From the apparently complicated expected volatility formula, this paper tries to simplify the VIX formula into an SEV Index to obtain new results about the information content in option prices. The simplified formulae for the expected volatility index are as follows:

$$SEV\_1 = \log(\Delta K) / \log(index),$$
$$SEV\_2 = \Delta K / index,$$
$$SEV\_3 = \Delta K / index^2,$$

where
ΔK = the difference between the strike prices.

From Figure 1 in the Appendix, we present graphs of the index, where the data start from 27 January 2008 through to 31 October 2008. Figure 2 illustrates each volatility index time series calculated from the above TVIX and SEV formulae. The summary statistics of the series are given in Table 1, as follows:

- The mean of the SEV_1 index is higher than those of SEV_3 and SEV_2, respectively, but lower than TVIX.
- From Figure 3 in the Appendix, all the indexes are positively skewed. The null hypothesis for the skewness coefficient that conforms to a normal distribution is zero, and this is rejected at the 5% significance level, with skewness coefficient greater than zero.
- All the indexes display kurtosis, or fat tails.

[Insert Table 1 around here]
[Insert Figure 2 around here]

6. Data

As TFEX index options are European-style, the basic Black-Scholes option pricing model is used, but it causes bias in the calculated implied volatility. Fleming *et al.* (1995) and Hull and White (1987) have found that the calculation of implied volatilities can eliminate the mis-measurement and bias problem from the near-the-money and close-to-expiry options. Therefore, a total of eight near-the-money close-to-expiry SET50 call and put options prices (four call options and four put options) are used to calculate expected volatility accurately.

Thus, VIX calculation represents the volatility of an hypothetical option that is at-the-money with a constraint 22 trading days (30-day calendar period) to expiration. However, TVIX calculation represents the volatility that is at-the-money with constraint 66 trading days (90-day calendar period) to expiration. For the SEV index, the trading days are used.

Both data series are obtained from Bloomberg (account at the Faculty of Economics, Chiang Mai University and Research Institute, Stock Exchange of Thailand). We obtain high-frequency intraday data, which are data at one-minute intervals between 09.45–12.30 and 14.30–16.55; for a total of 5 hours and 10 minutes each day. The sample period is from 27 January 2008 until 31 October 2008. The contract months are March, June, September, and December 2008. For contract month December 2008, the data are downloaded until 31 October 2008.

In order to estimate TVIX and SEV index and predict for call and put option price, we use the SAS 9.1 software package for the estimation and forecasting of time series data, as it offers a number of features that are not available in traditional econometric software.

As the SAS 9.1 software is used, the trading days for each month are counted through the actual trading days at the SET for SEV index since there is trading.

7. The Black-Scholes Model

The original Black and Scholes (1973) option-pricing model was developed to value options primarily on equities. The modified Black-Scholes European model that is used at the Thailand Futures Exchange (TFEX) has a number of restrictive assumptions, as follows:

1. The options pay no dividends during the option's life (q = 0);
2. European exercise terms dictate that the option can only be exercised on the expiration date;
3. Returns on the underlying asset are lognormally distributed;
4. No commissions are charged.

From the model given below, SET50 index call and put option prices are used to calculate implied volatility.

The TFEX Black-Scholes options pricing model is as follows:

<u>Call option pricing formula</u>:

$$C = Se^{-qt/365} \cdot N(d1) - Xe^{-rt/365} \cdot N(d2)$$

.

A call option affords the buyer the right to purchase an underlying asset for a fixed price in the future.

<u>Put options pricing formula</u>:

$$P = Xe^{-rt/365} \cdot (1 - N(d2)) - Se^{-qt/365} \cdot (1 - N(d1))$$

A put option affords the buyer the right to sell the underlying asset for a fixed price in the future:

$$d1 = \frac{\ln(S/X) + (r - q + (V^2/2)) \cdot (t/365)}{V \cdot \sqrt{t/365}}$$

$$d2 = d1 - V \cdot \sqrt{t/365}$$

where

S = price of underlying asset,

X = strike price at maturity date,

r = risk-free rate (apply zero-coupon bond at 3 month maturity to calculate options with 3 months maturity),

q = dividend yield of underlying asset (q = 0),

t = time to maturity (days),

N = the cumulative normal distribution function,

V = standard deviation of the rate of return during the life of the option (the expected volatility or TVIX).

With the Black-Scholes option pricing model, the expected volatilities are substituted to predict call and put option prices at each strike price and expiration.

8. Estimation

In order to assess the performance of the TVIX and SEV index, the model fit can be evaluated by measuring the descriptive statistics for the volatility index, as follows:

Measures of Statistic Fit	Equations
Mean Square Error	$\boldsymbol{MSE = \frac{SSE}{n}}$; $SSE = \sum_{t=1}^{n}(y_t - \hat{y})^2$, where $\overline{y}$ is the series mean.
Root Mean Square Error	$\boldsymbol{RMSE = \sqrt{MSE}}$
Mean Absolute Percent Error	$\boldsymbol{MAPE = \frac{100}{n}\sum_{t=1}^{n}\left\|(y_t - \hat{y}_t)/y_t\right\|}$
Mean Absolute Error	$\boldsymbol{MAE = \frac{1}{n}\sum_{t=1}^{n}\left\|\hat{y}_t - y_t\right\|}$
Adjusted R^2	$AdjR^2 = 1 - [(n-1)/(n-k)](1-R^2)$; $R^2 = 1 - SSE/SST$; $SST = \sum_{t=1}^{n}(y_t - \overline{y}_t)^2$ where n = the number of observations; p = the number of parameters including the intercept; i = 1 if there is an intercept, 0 otherwise
AIC	***n* ln(*MSE*) + 2 *k***
SBIC	***n* ln(*MSE*) + *k* ln(*n*)** where *k* is the number of estimated parameters

The mean square error (MSE) uses the one-step-ahead forecasts. Root mean square error (RMSE) is useful for determining how accurately the model might predict future observations. Adjusted R-squared (Adj R^2) is used as a standard model selection criterion. The Akaike information criterion (AIC) (Akaike (1973)) and Schwarz Bayesian Information criterion (SBIC) (Schwarz (1978)) are useful to determine which of several competing nested or non-nested models may fit the data the best. The model with the lowest values of AIC and SBIC is selected as fitting the sample data better.

[Insert Table 2 around here]

The value of adjusted R-squared closest to 1.00 indicates a good fit. The adjusted R-squared for SEV_1 is the highest, so SEV_1 is taken to be the best fitting model.

From Table 2, the AIC values of SEV_2, SEV_1 and TVIX exceed that of SEV_3, with 155,668; 183,217; and 433,372; respectively, so that the best fitting model is SEV_3, with the SEV_2 and SEV_1 models also providing better fits than the TVIX model.

Therefore, from the perspective of adjusted R-squared, AIC and SBIC, the

SEV model provides a better fit to the data than does the TVIX model.

[Insert Table 3 around here]

From Table 3, we compare each model across each quarter of the year as the quarterly contract month. In March and June 2008, the adjusted R-squared values of TVIX model are the closest to 1.00, but in September and December, the adjusted R-squared value of SEV_1 model is closest to 1.00.

Once again, the AIC and SBIC values of the SEV models are smaller than that of the TVIX model, so that the SEV models provide a better fit to the data.

The overall conclusion to be drawn is that, in terms of goodness of fit measures, our SEV index outperforms the formula used to calculate TVIX. For example, the RMSE of TVIX is larger than that of the SEV index.

[Insert Table 4 around here]
[Refer to Figure 4 around here]

From Table 4 and Figure 4 in the Appendix, we compare actual prices with the predicted prices from each model. In this case, selection of the best fitting model is not so clear, so we calculate the error between the actual and predicted prices.

[Insert Table 5 around here]
[Refer to Figure 5 around here]

Table 5 reports, and Figure 5 in the Appendix illustrates, the statistics relating to the errors. It can be seen that the mean of the error of the SEV_1 index is the lowest, and SEV_2 and SEV_3 have a lower range of errors compared with SEV_1 and TVIX. The errors of SEV_2 and SEV_3 are greater than the errors from SEV_1 and TVIX.

[Insert Table 6 around here]

From Table 6, the percentage error of TVIX is the least, followed by SEV_1, SEV_2 and SEV_3. Additionally, there is a high negative correlation between the SEV_1 index and the index over the year.

[Insert Table 7 around here]
[Insert Table 8 around here]

9. Conclusion

In this paper, we proposed a new and simplified volatility index, VIX, for expected volatility and pricing options from the seemingly complicated expected volatility formula established by the Chicago Board Options Exchange (CBOE). An extensive empirical analysis based on SET50 index options showed that the volatility index for Thailand, TVIX, provided more accurate predictions of option prices than the SEV index as the percent error is less. However, our simple expected volatility (SEV) index model outperformed TVIX in calculating and predicting expected volatility.

Our empirical results suggested that VIX is more accurate in formulating predictions. However, we also showed that the SEV index is more reliable than TVIX from the viewpoint of higher adjusted R-squared values, AIC and SBIC. Therefore, the SEV index would seem to be a superior tool as a hedging diversification tool, especially the SEV_1 index, because of the high negative correlation with the volatility index.

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Figure 1: Index time series

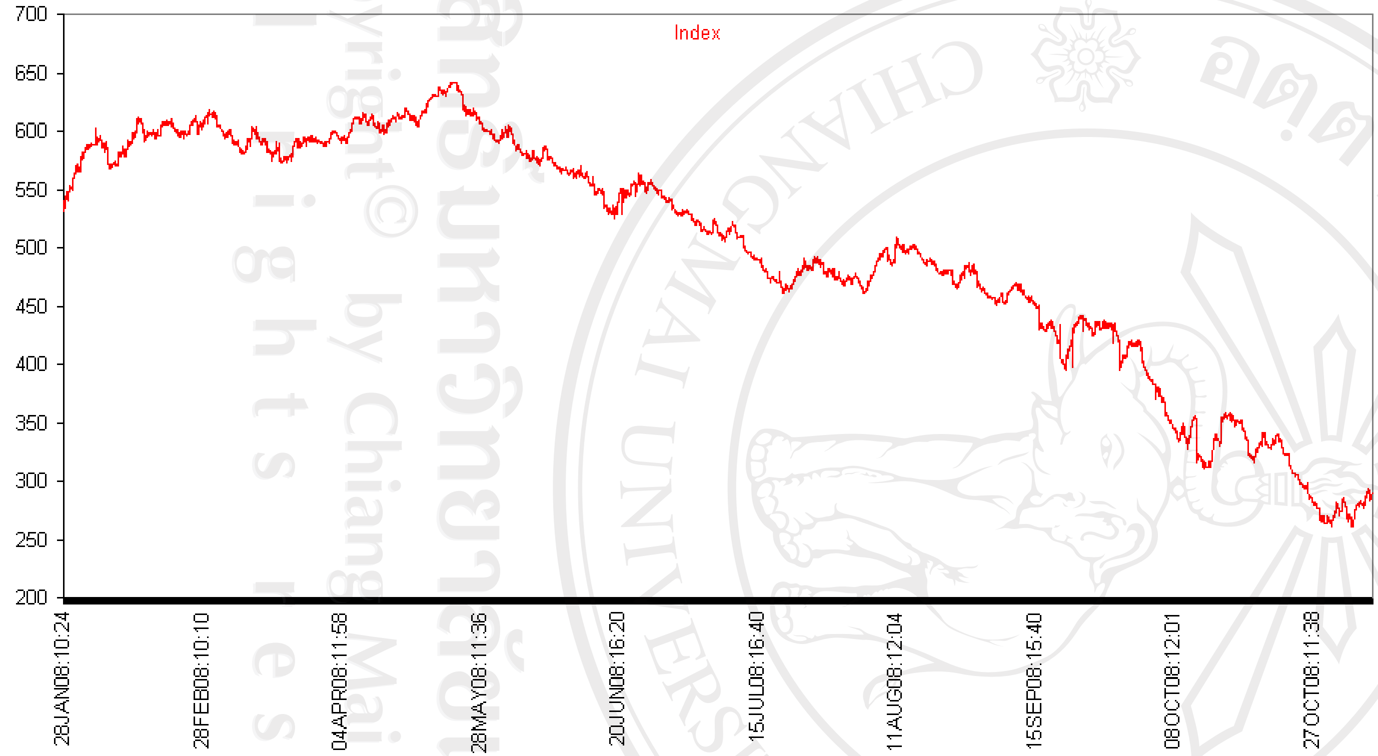

Figure 2: SEV index time series 27 January 2008 to 31 October 2008

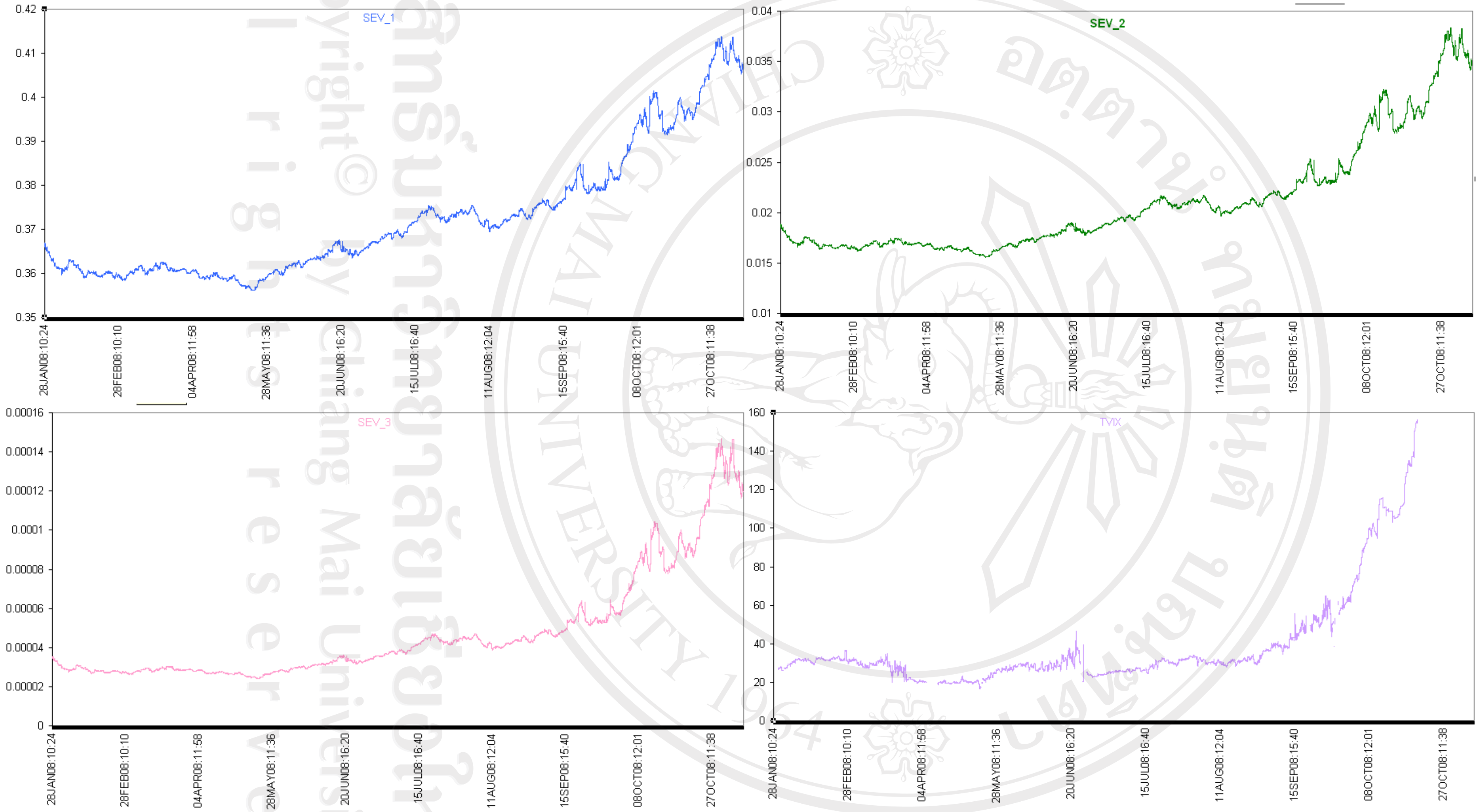

Figure 3: SEV index histogram

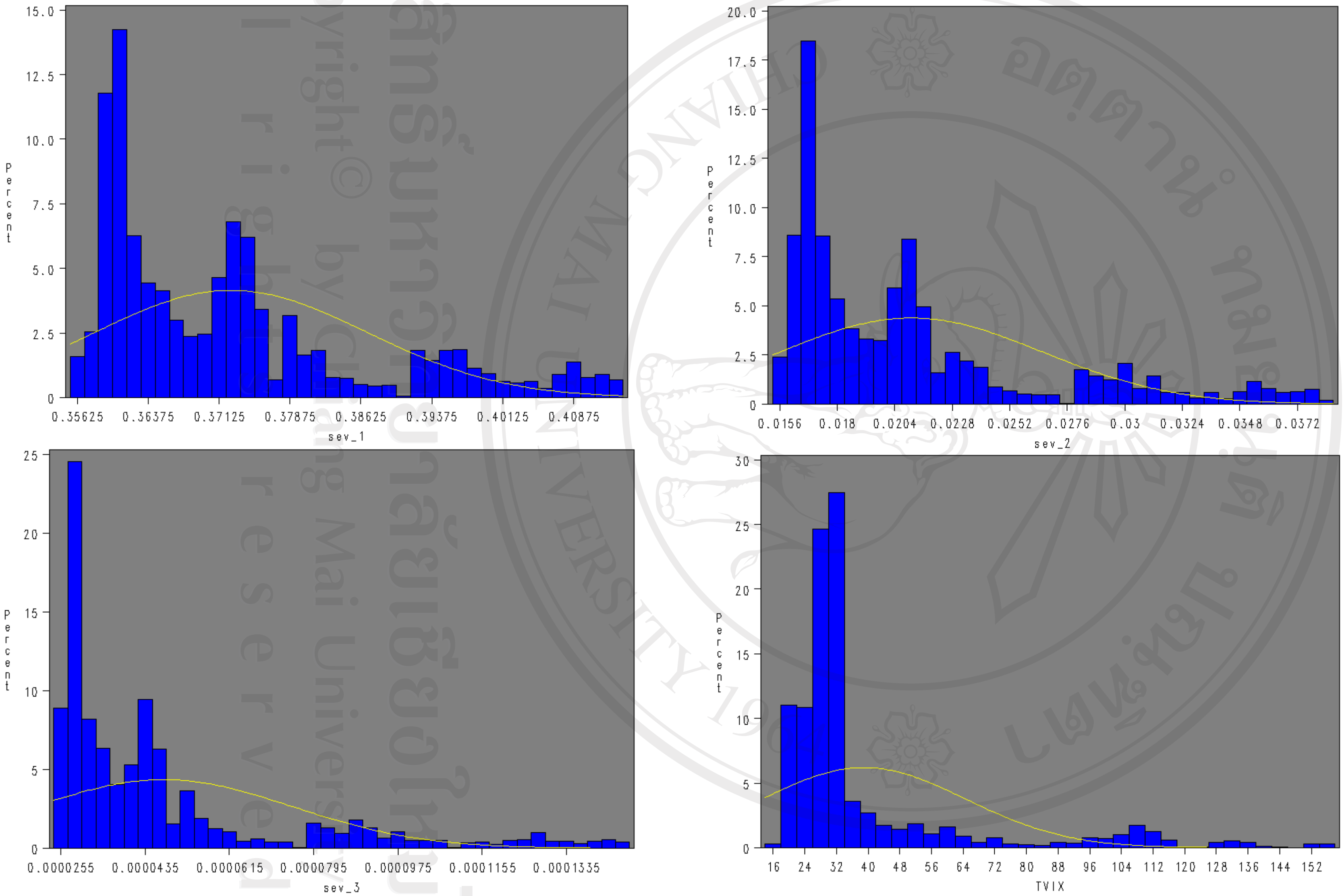

Figure 4: Actual price vs. predicted price by SEV index

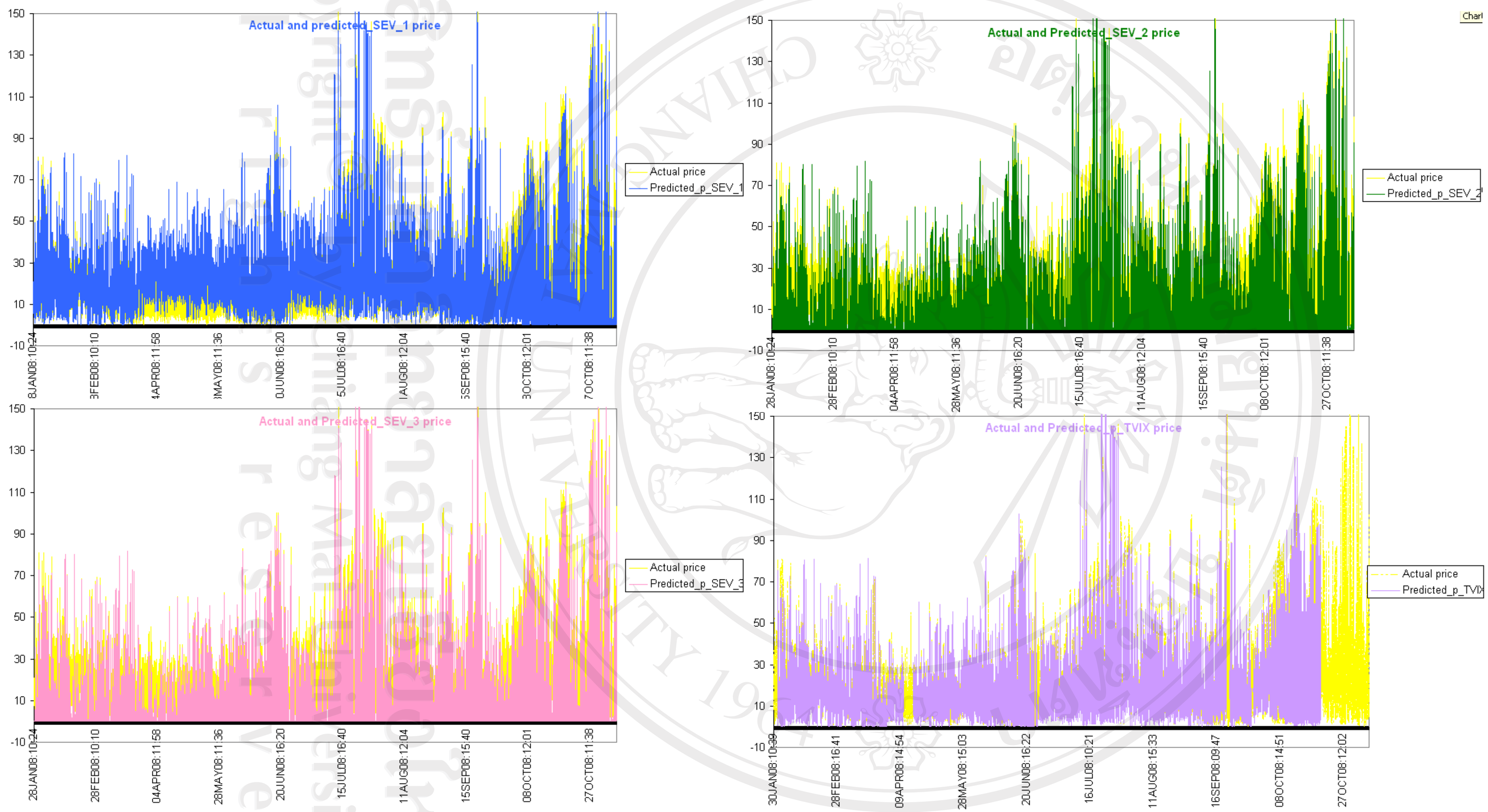

Figure 5: Error of SEV index

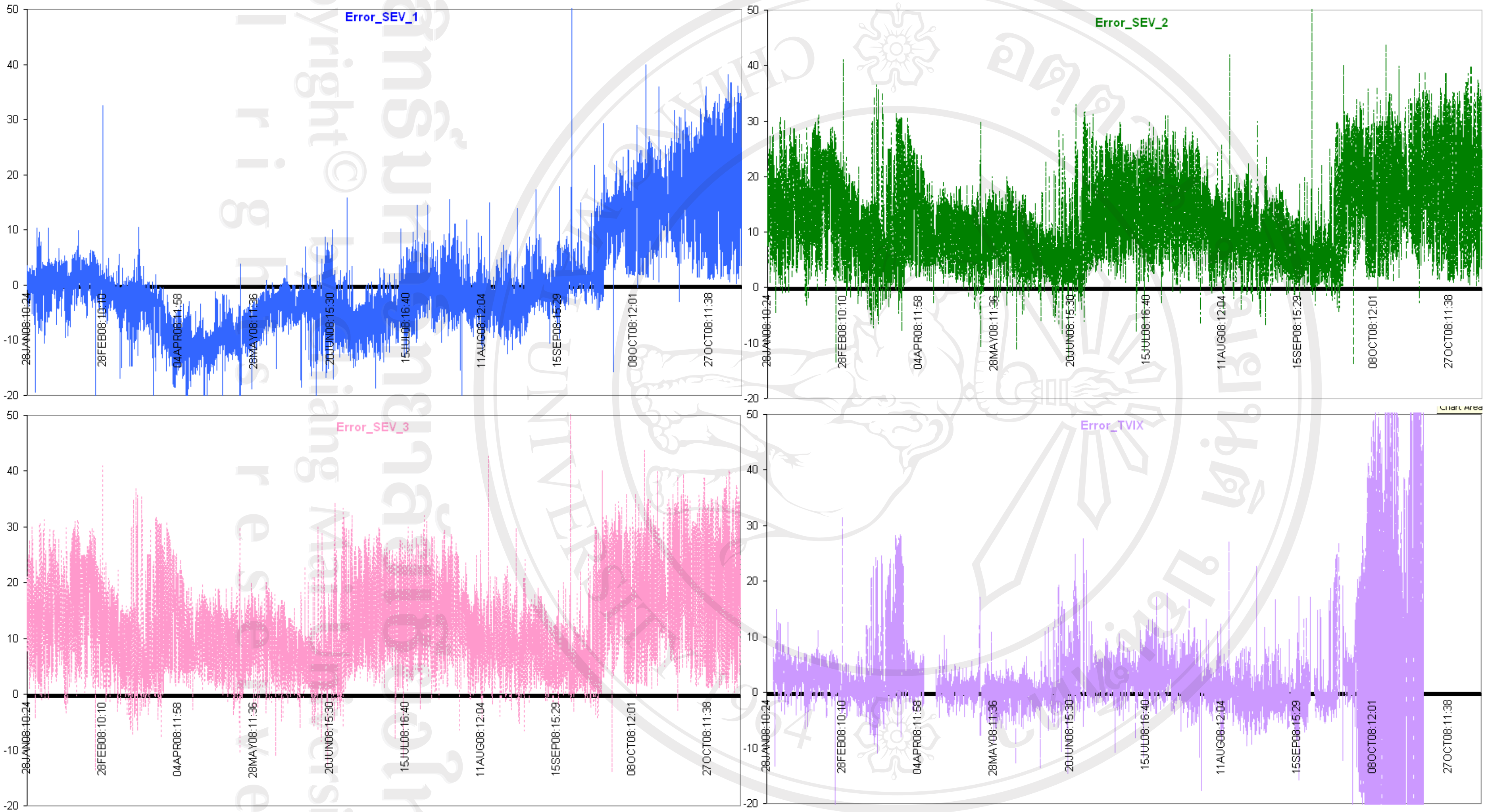

Figure 6: Error of Indexes

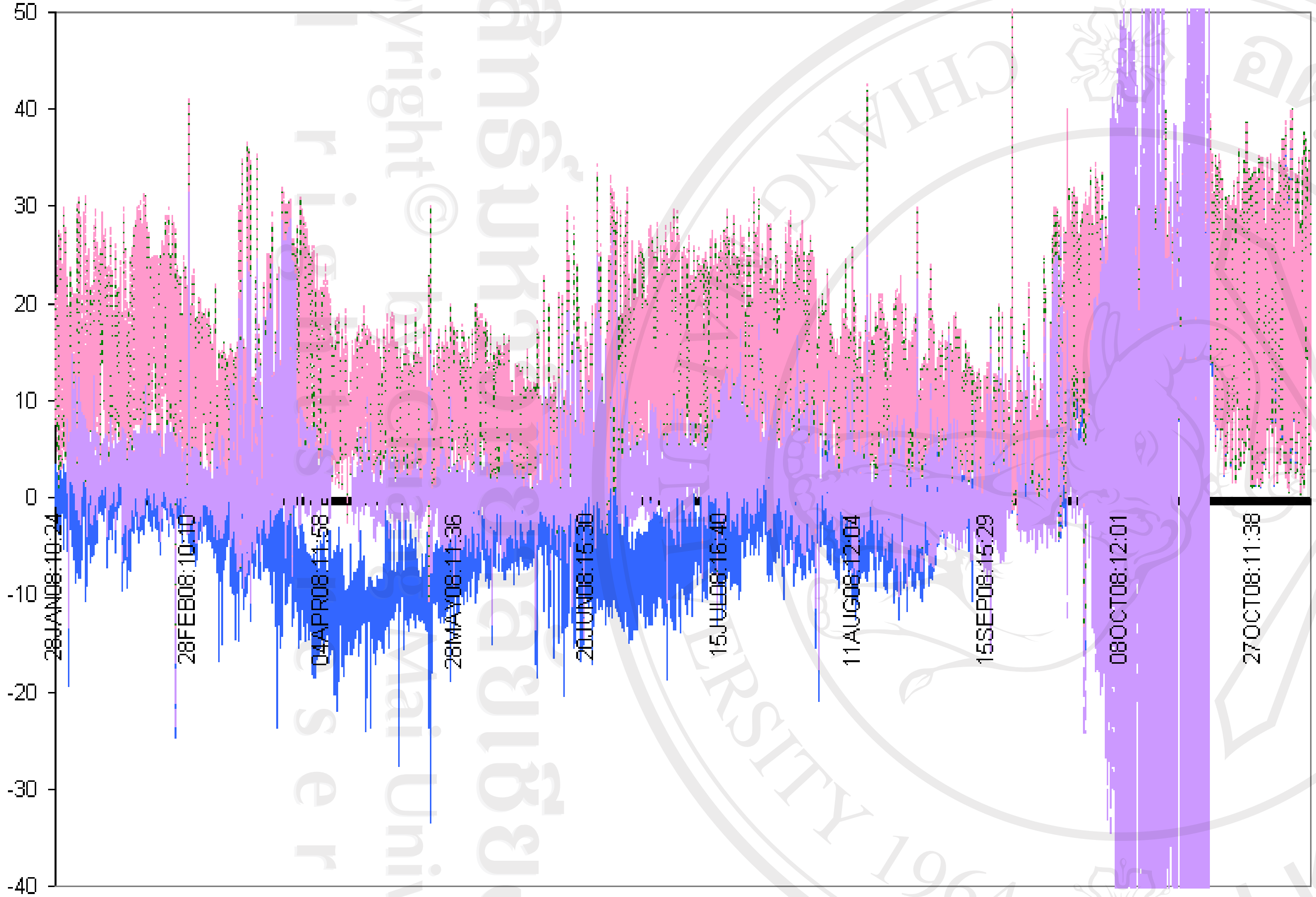

Table 1: Descriptive Statistics of Volatility Indexes

Variable	SEV_1	SEV_2	SEV_3	TVIX
Mean	0.37	0.02	0.000048	38.98
Std Dev	0.01	0.01	0.000028	25.76
Kurtosis	1.18	1.43	1.81	2.42
Skewness	0.46	1.19	2.57	5.24
Minimum	0.36	0.02	0.000024	16.60
Maximum	0.41	0.04	0.000147	156.75

Table 2: Goodness of Fit of Volatility Indexes

Measures of Goodness of Fit	SEV_1	SEV_2	SEV_3	TVIX
Mean Square Error MSE	2.30E-08	3.84E-09	1.33E-07	0.38068
Root Mean Square Error RMSE	0.0001516	0.0000620	0.0003648	0.6169900
Mean Absolute Percent Error MAPE	**0.01503**	0.09208	0.18429	0.50004
Mean Absolute Error MAE	0.0000566	0.0000207	0.0001003	0.1754700
Adjusted R-Square R^2 (Close to 1.000)	**0.99989**	0.99987	0.99982	0.99943
AIC	-262,874	-290,423	**-446,091**	-12,718
SBIC	-262,851	-290,400	**-446,068**	-12,704

Table 3: Summary Statistics Over the Year

March 2008	SEV_1		SEV_2		SEV_3		TVIX	
	Call	Put	Call	Put	Call	Put	Call	Put
MSE	2.31E-08	2.25E-08	2.09E-09	2.04E-09	2.45E-14	2.39E-14	0.61369	0.66007
RMSE	0.0001519	0.00015	0.0000457	0.0000452	1.56E-07	1.55E-07	0.78338	0.81245
MAE	0.0000786	0.0000771	0.0000236	0.0000231	8.05E-08	7.87E-08	0.26397	0.27636
MAPE	0.02176	0.02136	0.13874	0.13632	0.27782	0.27277	7.54E-01	0.77285
ADJ R^2	0.98919	0.98712	0.98933	0.98722	0.98952	0.98734	**0.99886**	**0.99914**
AIC	-26762.31	-25954.18	-30416.3	-29491.16	**-47700.98**	**-46228.35**	-2823.69	-2837.35
SBIC	-26756.98	-25948.88	-30410.97	-29485.86	**-47695.65**	**-46223.05**	-3074.72	-2823.69

June 2008	SEV_1		SEV_2		SEV_3		TVIX	
	Call	Put	Call	Put	Call	Put	Call	Put
MSE	2.26E-08	2.13E-08	2.10E-09	1.98E-09	2.55E-14	2.41E-14	0.61369	0.66007
RMSE	0.0001504	0.0001459	0.0000458	0.0000445	1.60E-07	1.55E-07	0.78338	0.81245
MAE	0.0000748	0.0000718	0.0000226	0.0000217	7.74E-08	7.53E-08	0.26397	0.27636
MAPE	0.02069	0.01987	0.13185	0.12659	0.26376	0.2557	0.7537	0.77285
ADJ R^2	0.99554	0.99615	0.99546	0.99611	0.99531	0.99602	**0.99886**	**0.99914**
AIC	-59374.92	-61949.31	-67393.37	-70282.14	**-105568**	**-109968**	-3088.22	-2837.35
SBIC	-59362.67	-61936.99	-67381.13	-70269.82	**-105556**	**-109955**	-3074.72	-2823.69

September 2008	SEV_1		SEV_2		SEV_3		TVIX	
	Call	Put	Call	Put	Call	Put	Call	Put
MSE	3.62E-08	2.95E-08	4.35E-09	3.44E-09	8.20E-14	6.17E-14	0.61369	0.66007
RMSE	0.0001901	0.0001719	0.0000659	0.0000586	2.86E-07	2.48E-07	0.78338	0.81245
MAE	0.0000811	0.0000781	0.0000265	0.0000256	1.04E-07	9.98E-08	0.26397	0.27636
MAPE	0.02206	0.02124	0.13808	0.13343	0.2762	0.26596	0.61369	0.77285
ADJ R^2	**0.99923**	**0.99945**	0.99914	0.99942	0.99892	0.99932	0.99886	0.99914
AIC	-104093	-107888	-116962	-121276	**-183050**	**-189272**	-3088.22	-2837.35
SBIC	-104080	-107874	-116949	-121262	**-183030**	**-189252**	-3074.72	-2823.69

December 2008	SEV_1		SEV_2		SEV_3		TVIX	
	Call	Put	Call	Put	Call	Put	Call	Put
MSE	6.02E-08	4.89E-08	9.85E-09	8.07E-09	3.30E-13	2.76E-13	0.61369	0.66007
RMSE	0.0002453	0.0002212	0.0000992	0.0000898	5.74E-07	5.25E-07	0.78338	0.81245
MAE	0.0000978	0.0000925	0.0000355	0.0000342	1.71E-07	1.68E-07	0.26397	0.27636
MAPE	0.02597	0.02448	0.15884	0.1499	0.31797	0.29911	0.7537	0.77285
ADJ R^2	**0.99968**	**0.99978**	0.99963	0.99975	0.99951	0.99967	0.99886	0.99914
AIC	-119189	-119175	-132163	-132149	**-206023**	**-206002**	-3088.22	-2837.35
SBIC	-133260	-133246	-147531	-147517	**-228953**	**-228932**	-3074.72	-2823.69

Table 4: Summary of Actual and Predicted Prices

	Actual Price		Predicted Prices							
Variable	C_Price	P_Price	C_SEV_1	P_SEV_1	C_SEV_2	P_SEV_2	C_SEV_3	P_SEV_3	C_TVIX	P_TVIX
Mean	15.09	24.39	21.79	28.52	10.04	16.77	10.18	16.91	17.94	17.60
Std Dev	10.75	21.15	17.33	23.24	16.51	25.35	16.45	25.27	16.73	17.06
Minimum	0.1	0	2.87E-100	-1.89E-14	0	-8.3E-14	0	-7.9E-14	5.02E-26	2.02E-07
Maximum	80.00	210.00	130.15	208.69	130.05	208.69	130.05	208.69	130.23	208.76

Note: C and P denote call and put, respectively.

Table 5: Summary Statistics of Forecast Errors

	Error = actual price – predicted price			
Variable	**error_SEV_1**	**error_SEV_2**	**error_SEV_3**	**error_TVIX**
Mean	**0.45**	12.20	12.26	1.40
Std Dev	9.46	7.85	7.90	11.84
Minimum	-33.45	-13.97	-13.97	-96.23
Maximum	58.31	58.31	58.31	71.63
Sum	**6800.74**	184041.61	184891.54	18475.16

Table 6: Summary Statistics of Percentage Errors

Variable	**%_error_SEV_1**	**%_error_SEV_2**	**%_error_SEV_3**	**%_error_TVIX**
Mean	-18.35	80.40	80.71	**-3.47**
Std Dev	101.50	32.34	32.47	91.76
Minimum	-2367.00	-131.22	-131.22	-1762.51
Maximum	100	100	100	100

Table 7: Correlations between the Volatility Indexes and Index

Correlations	SEV_1	SEV_2	SEV_3	TVIX
Index	**-0.96462**	-0.95044	-0.92128	-0.66855

Correlations	SEV_1	SEV_2	SEV_3	TVIX
Index				
March 2008	**-0.99958**	-0.99909	-0.99798	0.31342
June 2008	**-0.99936**	-0.99855	-0.99681	-0.48684
September 2008	**-0.99765**	-0.99450	-0.98722	-0.68363
December 2008	**-0.96462**	-0.95044	-0.92128	-0.66855

Table 8: Summary of Criteria for Best Fitting Models

Criteria	SEV_1	SEV_2	SEV_3	TVIX
MSE				
RMSE				
MAE				
MAPE				
Adjusted R^2				
AIC				
SBIC				
Error				
Percent error				
APM				
Correlations				

APPENDIX B

ARFIMA-FIGARCH and ARFIMA -FIAPARCH

on Thailand Volatility Index

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ARFIMA-FIGARCH and ARFIMA -FIAPARCH on Thailand Volatility Index*

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ABSTRACT

This paper applied SET50 Index options with the Chicago Board Options Exchange (CBOE) as a Thailand Volatility Index (TVIX). This can be considered as a hedging diversification tool because of the high negative correlation with stock index. In addition, we estimate ARFIMA-FIGARCH and ARFIMA-FIAPARCH which are capable of capturing long memory and asymmetry in the conditional variance and power transformed conditional variance of process. The empirical shows that the best model with accuracy is ARMA-FIAPARCH.

1. Introduction

Volatility Index (VIX) measures market expectation of near term volatility conveyed by stock index option prices. The original VIX was constructed using the implied volatilities of eight different S&P100 (OEX) option series so that, at any given time, it represented the implied volatility of a hypothetical at-the-money OEX option with exactly 30 days to expiration from an option-pricing model.

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In 1993, the Chicago Board Options Exchange (CBOE) introduced the CBOE Volatility Index, VIX, which quickly became the benchmark for stock market volatility.

In 2003, the CBOE made two key enhancements to the VIX methodology. The new VIX is based on an up-to-the-minute market estimation of expected volatility that is calculated by using real-time S&P500 Index (SPX) option bid/ask quotes. Until 2006, VIX was trading on the CBOE. The VIX options contract is the first product on market volatility to be listed on an SEC-regulated securities exchange. This new product can be traded from an options-approved securities account. Many investors consider the VIX to be the world's premier barometer of investor sentiment and market volatility, and VIX option is a very powerful risk management tool.

For the econometric model, it is assumed to be ceteris paribus with the variance and error term as constant terms. ARCH (Autoregressive Conditional Heteroscedastic) was developed and applied to ARMA (Autoregressive Moving Average) model in order to correct the assumption contradiction of time series economics data. The data has high variance with non-stationary variance and error term (Enders, 1995). The investors simply focus on the conditional variance such as the prediction of return and variance. ARMA shows the value of mean and variance simultaneously (Engle, 1982). GARCH (Generalized Autoregressive Conditional Heteroscedastic) model was developed from ARCH in order to adjust the variance to characterize as ARMA process which is applied in time variance model in money market (Engle, 1982 and Bollerslev, 1986).

Baillie, Bollerslev, and Mikkelsen (1996) proposed FIGARCH (Fractional Integrated GARCH) model which is effectively capture both volatility clustering and long memory because GARCH model exhibit short memory and cannot analyze hyperbolic memory in conditional volatility process and capture asymmetries in equity market volatility. With the previous findings of Ding, Granger, and Engle (1993) and Baillie *et al.* (1996) (among others) who suggest the modeling of conditional variance of high frequency financial data by the use of an (Asymmetric) Power GARCH (APARCH) or Fractionally Integrated GARCH (FIGARCH) models. Tse (1998) develops the Fractionally Integrated Asymmetric Power ARCH (FIAPARCH) model, which allows for long memory and asymmetries in volatility.

This paper is to calculate TVIX basing on SET50 index options with CBOE VIX formula with the nearest-month contracts and compare the best performance of ARFIMA-FIGARCH and -FIAPARCH models for forecasting TVIX.

2. Volatility Index

Estimating implied volatility from options is no straightforward method to extract the information. Whaley (2000) considered implied volatility as a fear gauge because option prices calculate implied volatility that represents a market-based estimate of future price volatility). Implied volatilities are the information by investors, financial news services and other finance professionals. The information content and forecast quality of implied volatility is an important topic in financial markets research.

Latane and Rendleman (1976), Chiras and Manaster (1978), Beckers (1981) and Jorion (1995) provided early assessments of the forecast quality of implied volatility and concluded that implied volatility outperforms historical standard deviations and is a good predictor of future volatility, although it might be biased. Christensen and Prabhala (1998) also found that implied volatility forecasts are biased, but dominate historical volatility in terms of ex ante forecasting power. Fleming (1998) used a historical volatility measure to show that implied volatilities outperform historical information.

Dennis *et al.* (2006) found that daily innovations in VIX contain very reliable incremental information about the future volatility of the S&P100 index. Other studies that attempt to forecast implied volatility or use the information contained in implied volatility to trade in option markets include Harvey and Whaley (1992), Noh *et al.* (1994), and Poon and Pope (2000).

3. New VIX Procedure

The New VIX is more robust because it pools the information from option prices over the whole volatility skew, and not just from at-the-money options. The formula used in the new VIX calculation is given by the CBOE as follows:

$$\sigma^2 = \frac{2}{T}\sum_i \frac{\Delta K_i}{K_i^2} e^{RT} Q(K_i) - \frac{1}{T}\left[\frac{F}{K_0} - 1\right]^2$$

where

σ = VIX / 100 (so that VIX = σ x 100),

T = Time to expiration (in minutes),

F = Forward index level, derived from index option prices (based on at-the-money option prices, the difference between call and put prices is smallest).

The formula used to calculate the forward index level is:

F = Strike price (at-the-money) + e^{RT} x (Call price – Put price),

where

R = risk-free interest rate is assumed to be 3.01% (for simplicity, the government T-bills 3 month contract interest rate is used, as the Thailand options contract is a 3 months contract);

T = $\{M_{current\ day} + M_{settlement\ day} + M_{other\ days}\}$/minutes in a year,

where

$M_{current\ day}$ = # of minutes remaining until midnight of the current day,

$M_{settlement\ day}$ = # of minutes from midnight until 9:45 am on the TFEX settlement day,

$M_{other\ days}$ = Total # of minutes in the days between the current day and the settlement day;

K_i = Strike price of i^{th} out-of-the-money option; a call if $K_i > F$ and a put if $K_i < F$;

ΔK_i = Interval between strike prices - half the distance between the strike on either side of K_i: $\Delta K_i = \frac{K_{i+1} - K_{i-1}}{2}$.

K_0 = First strike below the forward index level, F;

$Q(K_i)$ = The midpoint of the bid-ask spread for each option with strike K_i.

(Note: ΔK_i for the lowest strike is simply the difference between the lowest strike and the next higher strike. Likewise, Δ K for the highest strike is the difference between the highest strike and the next lower strike.)

With the adaptation of the VIX calculation to Thailand SET 50 index options, the Thailand expected volatility (TVIX) can be estimated.

4. Theory

4.1 ARFIMA Model

ARIMA models are frequently used for seasonal time series (Box and Jenkins, 1976). A general multiplicative seasonal ARIMA model for time series Z_t is as follows:

$$\phi(L)\Phi(L^s)(1-L)^d(1-L^s)^D Z_t = \theta(L)\rho(L^s)a_t$$

where:

L = a backshift or lag operator $(B_{zt} - Z_{t-1})$

S = seasonal period

$\phi(L)$ = $(1-\phi_1 L - ... - \phi_p L^p)$ is the non-seasonal AR operator

$\Phi(L^s)$ = $(1-\Phi_1 L^s - ... - \Phi_s L^s)$ is the seasonal AR operator

$\theta(L)$ = $(1-\theta_1 L - ... - \theta_q L^q)$ is the non-seasonal MA operator

$\rho(L)$ = $(1-\rho_1 L^s - ... - \theta_0 L^{Q_s})$ is the seasonal MA operator

$(1-L)^d(1-L^s)$ = non-seasonal differencing of order d and seasonal differencing of order D

Granger and Joyeux (1980) and Hosking (1981) proposed an autoregressive fractionally integrated moving-average (ARFIMA) model and proposed the method to fit long-memory data. ARFIMA(p,d,q) is written as follow:

$$\phi(L)\Delta^d y_t = \delta + \theta(L)u_t$$

with:

$$\phi(L) = 1-\phi_1 L - ... - \phi_p L^p \text{ and}$$

$$\theta(L) = 1-\theta_1 L - ... - \theta_q L^q$$

where:

δ = a constant term

$\theta(L)$ = the MA operator at order q

u_t = an error term

$\phi(L)$ = the AR operator at order p

$\Delta^d y_t$ = the differencing operator at order d of time series data y_t

For d = *(-0.5, 0)*, the process exhibits intermediate memory or long range negative dependence, while d = *(0, 0.5)*, the process exhibits long memory or long range positive dependence. For d = *[0.5, 1)*, the process is mean reverting with no long run impact to future values of the process and the process becomes a short memory when d = 0 corresponding to a standard ARMA process.

4.2 FIGARCH Model

The GARCH model by Bollerslev (1986) imposed important limitations, not to capture a positive or negative sign of u_t, which both positive and negative shocks has the same impact on the conditional variance, h_t, as follows,

$$u_t = \eta_t \sqrt{\sigma_t},$$

$$\sigma_t^2 = \omega + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 + \sum_{i=1}^{q} \alpha_i u_{t-i}^2$$

where $\omega > 0$, $\alpha_i \geq 0$ for i = 1,…,p and $\beta_j \geq 0$ for j = 1,…,q are sufficient to ensure that the conditional variance, h_t, is non-negative. For the GARCH process to be defined, it is required that $\omega > 0$. Therefore, a univariate GARCH(1,1) model is known as ARCH(∞) model (Engle, 1982) as an infinite expansion in u_{t-i}^2.

Baillie *et al.* (1996) proposed fractionally integrated GARCH (FIGARCH) model to determine long memory in return volatility. The FIGARCH (p,d,q) process is as follow:

$$\phi(L)(1-L)^d u_t^2 = \omega + [1-\beta(L)]v_t ,$$

where $v_t = u_t^2 - \sigma_t^2$, $0 < d < 1$, $\phi(L) = \sum_{i=1}^{m-1} \phi_i L^i$ is of order m-1, and all the roots of $\phi(L)$ and $[1-\beta(L)]$ lie outside the unit circle. The FIGARCH model is derived from standard GARCH model with fractional difference operator, *(1- L)*^{d}. The FIGARCH(p,d,q) model is reduced to the standard GARCH when d = 0 and becomes IGARCH model when d = 1.

Baillie *et al.* (1996) claimed with the arguments of Nelson (1990) that the FIGARCH(p,d,m) is ergodic and strictly stationary which is difficult to verify. The degree of persistence of the FIGARCH model operates reversely direction of the ARFIMA process.

Chung (2001) suggested the analysis of the FIGARCH specification

$$\sigma_t^2 = \{1-[1-\beta(L)]^{-1})(1-L)^d \phi(L)\}\varepsilon_t^2$$

4.3 FIAPARCH Model

Tse (1998) extended the asymmetric power ARCH (APARCH) model of Ding *et al.* (1993) to fractionally integrated of Baillie *et al.* (1996) which is extended to FIAPARCH model as follows:

$$\sigma_t^\delta = \omega + \left[1 - \frac{[1-\phi(L)](1-L)^d}{1-\beta(L)})\right]\left[|u_t| - \gamma u_t\right]^\delta$$

where $0 < d < 1$, $\omega, \delta > 0, \phi, \beta < 1$, $-1 < \gamma < 1$ and L is the lag operator. When $\gamma > 0$, negative shocks have a higher volatility than positive shocks. The particular value of power term may lead to suboptimal modeling and forecasting performance. Ding *et al.* (1993) found that the closer of d value converge to 1, the larger the memory of the process becomes. The process of FIAPARCH allows for asymmetry. When $\gamma = 0$ and $\delta = 2$, the process of FIAPARCH is reduced to FIGARCH process.

ARFIMA-FIAPARCH generates the long memory property in both the first and (power transformed) second conditional moments and is sufficiently flexible to handle the dual long memory behavior. It can recognize the long memory aspect and provides an empirical measure of real uncertainty that accounts for long memory in the power transformed conditional variance of the process.

5. Data Descriptive

One-minute intervals of SET50 Index options are obtained from Bloomberg accounted by the Faculty of Economics, Chiang Mai University and Research Institute, Stock Exchange of Thailand. The sample period is from 27 January 2008 until 30 September 2009. The contract months are March, June, September 2008 and 2009 and December 2008.

In order to calculate TVIX, we use the SAS 9.1 software package for the calculation as it offers a number of features that are not available in traditional econometric software. Also, OxMetrics5 software is used to estimate ARFIMA-FIGARCH and -FIAPARCH on daily returns.

The returns of TVIX at time t are calculated as follows:

$$\boldsymbol{R_{i,t} = \log(P_{i,t} / P_{i,t-1})}$$

where $\boldsymbol{P_{i,t}}$ and $\boldsymbol{P_{i,t-1}}$ are the closing prices of TVIX at time t and t-1, respectively.

[Insert Table 1 around here]

Table 1 presents the descriptive statistics for the returns of TVIX. The average return of TVIX is negative. The normal distribution has a skewness statistic equal to zero and a kurtosis statistic of 3, but return of TVIX has negative skewness statistics and high kurtosis, suggesting the presence of fat tails. This means that the data has a longer left tail (extreme losses) than right tail (extreme gain). Figure 1 presents the plot of TVIX and TVIX returns. This indicates some circumstances where TVIX returns fluctuate.

Table 2 summarized the unit root tests for TVIX returns. The Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests were used to test the null hypothesis of a unit root against the alternative hypothesis of stationarity. The tests yield large negative values in all cases for levels such that the individual returns series reject the null hypothesis at the 1% significance level, hence, the returns are stationary.

[Insert Figure 1 around here]
[Insert Table 2 around here]

6. Estimation

The Akaike Information Criterion (AIC) (Akaike, 1973) and Schwarz Bayesian Information Criterion (SBIC) (Schwarz, 1978) are useful to determine the best fit among several competing nested or non-nested models The model with the lowest values of AIC and SBIC is selected as fitting the sample data better.

[Insert Table 3 around here]

Table 3 shows forecasting method based on ARFIMA-FIGARCH and ARFIMA-FIAPARCH models for forecasting TVIX. The values of both AIC and SBIC in each of ARFIMA-FIGARCH and ARFIMA-FIAPARCH model are used for selection the best ARFIMA-FIGARCH and ARFIMA-FIAPARCH models for forecasting TVIX for this period.

The lowest values of AIC and SBIC are 3.882 and 35.466, respectively for ARFIMA$(1, d_m, 1)$-FIGARCH$(1, d_v, 1)$ and 5.848 and 41.381, respectively for ARFIMA$(1, d_m, 1)$-FIAPARCH$(1, d_v, 1)$.

For ARFIMA$(1, d_m, 1)$-FIGARCH$(1, d_v, 1)$ and ARFIMA$(1, d_m, 1)$-FIAPARCH$(1, d_v, 1)$, d_m for ARFIMA$(1, d_m, 1)$ is -0.0857 and -0.0901, respectively. For ARFIMA$(3, d_m, 3)$-FIGARCH$(1, d_v, 1)$ and ARFIMA$(3, d_m, 3)$-FIAPARCH$(1, d_v, 1)$, d_m of ARFIMA is -0.0491 and -0.0475, respectively. All d_m of ARFIMA processes are not statistically significant. This can be concluded that the process is ARMA-

FIGARCH and ARMA-FIAPARCH. Both processes are intermediate memory or long range negative dependence for TVIX.

The estimations of both d_v for ARFIMA$(1, d_m, 1)$-FIGARCH$(1, d_v, 1)$ and ARFIMA$(3, d_m, 3)$-FIGARCH$(1, d_v, 1)$ are 0.5397 and 0.5568 which are more than 0.5 at 1% level of significance. This means that both processes are statistically significant long memory and can be estimated in long run. However, the estimations of both d_v for ARFIMA$(1, d_m, 1)$-FIAPARCH$(1, d_v, 1)$ and ARFIMA$(3, d_m, 3)$-FIAPARCH$(1, d_v, 1)$ are 0.3786 and 0.3950 which are less than 0.5 at 1% level of significance. This means that both processes are statistically significant short memory and cannot be estimated in long run. FIAPARCH is not long memory because d_v is less than 0.5.

Hence, ARFIMA$(3, d_m, 3)$-FIGARCH$(1, d_v, 1)$ has the largest memory than others which can be estimated in long run.

From table 4, mean absolute error (MAE) and mean absolute percent error (MAPE) of all models show that ARFIMA$(3, d_m, 3)$-FIGARCH$(1, d_v, 1)$ is fitted in forecasting returns of TVIX as the lowest of both values.

Consequently, with the lowest AIC and SBIC, ARFIMA(1,-0.0857,1)-FIGARCH(1,0.5397,1) and ARFIMA(1,-0.0910,1)-FIAPARCH(1,0.3786,1) models are fitted to the data. However, comparing both processes, ARFIMA(1,-0.0857,1)-FIGARCH(1,0.5397,1) is better than ARFIMA(1,-0.0910,1)-FIAPARCH(1,0.3786,1). ARFIMA(3,-0.0491,3)-FIGARCH(1,0.5568,1) model provides the best fit to the data with the lowest of MAE and MAPE in forecasting and has the largest memory than others which can be estimated in long run.

However, the process of ARFIMA is not statistically significant. Therefore, the null hypothesis of ARFIMA is accepted. The process becomes a short memory corresponding to a standard ARMA process. ARMA-FIGARCH and ARMA-FIAPARCH are estimated as follows:

[Insert Table 4 around here]
[Insert Figure 2 around here]
[Insert Table 4 around here]
[Insert Table 5around here]

From table 5, the AIC and SBIC criteria values strongly favor the ARMA-FIGARCH formulation over the ARMA-FIAPARCH. However, the value of d_v for ARMA-FIAPARCH is greater than FIGARCH with statistically significant at the 1% level. This means that the ARMA-FIAPARCH process is statistically significant longer memory and can be estimated in long run than ARMA-FIGARCH, also with the larger statistically significant value of power term. Moreover, positive shocks have a higher volatility than negative shocks as $\gamma < 0$. This implies that positive shocks on TVIX are negative shocks in index options because TVIX and SET50 Index options are oppositely correlated.

7. Conclusion

This paper applies SET50 Index options with the CBOE volatility index, VIX, formulae as a TVIX which is the benchmark for stock market volatility and leveraging, whereby leverage allows traders to make a significant amount of money from a relatively small change in price.

We also analyze ARFIMA-FIGARCH - FIAPARCH models for the best prediction for returns of TVIX. From the viewpoint of AIC and SBIC, ARFIMA$(1, d_m, 1)$-FIGARCH$(1, d_v, 1)$ is the best fit for modeling. However, from the viewpoint of MAE, MAPE and d_v, our empirical results show that ARFIMA$(3, d_m, 3)$-FIAPARCH$(1, d_v, 1)$ is the best accuracy in forecasting returns of TVIX with the largest memory.

Moreover, with the statistically significant of d_m and d_v, the process becomes ARMA-FIGARCH and ARMA-FIAPARCH for the data. Therefore, both processes are also estimated. The results show that ARMA-FIGARCH is better fit to the data by using AIC and SBIC criteria values, however, ARMA-FIAPARCH is longer memory than ARMA-FIGARCH and capture asymmetric effect.

However, both the Stock Exchange of Thailand and Security Exchange Commission should firstly develop and launch TVIX as a hedging diversification tool in the market in order, for the investors, to learn and be acquainted with TVIX for a few years primarily, and apply the forecasted model to forecast TVIX.

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Table 1: Descriptive Statistics of TVIX Returns

	Mean	Std Dev	Skewness	Kurtosis	Max	Min
TVIX	-0.00022	0.093806	-1.08530	11.41235	0.44728	-0.52266

Figure 1: Daily TVIX and Returns

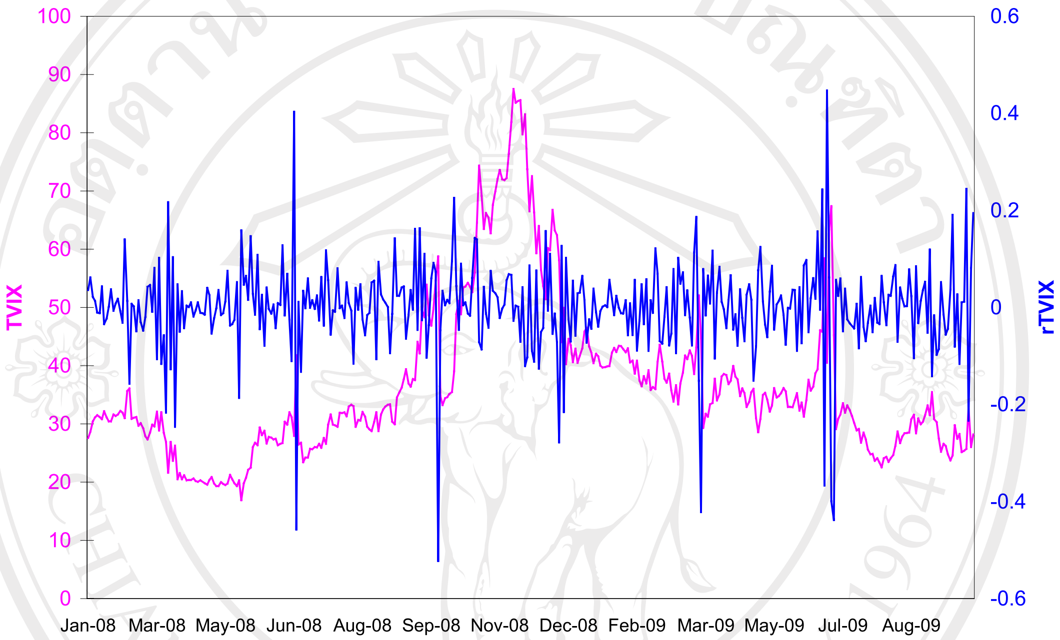

Table 2: Unit Root Test for Returns of TVIX

	ADF Test			Phillips-Perron Test		
Returns	**None**	**Constant**	**Constant and Trend**	**None**	**Constant**	**Constant and Trend**
TVIX	-24.022*	-23.991*	-23.977*	-24.560*	-24.526*	-24.522*

Note: * significant at the 1% level.

Table 3: Accuracy comparison in sample for different forecasting models of ARFIMA-FIGARCH and ARFIMA-FIAPARCH based on concept of both AIC and SBIC criterion

ARFIMA-FIGARCH	AIC	SBIC
ARFIMA(1,d,1)-FIGARCH(1,d,1) d of ARFIMA = -0.0857, d of FIGARCH = 0.5397* (0.2509) (0.0000)	3.882	35.466
ARFIMA(1,d,2)-FIGARCH(1,d,1) d of ARFIMA = -0.1367, d of FIGARCH = 0.5435* (0.4902) (0.0000)	5.883	41.416
ARFIMA(1,d,3)-FIGARCH(1,d,1) d of ARFIMA = -0.0331, d of FIGARCH = 0.5336* (0.7817) (0.0000)	7.880	47.360
ARFIMA(2,d,3)-FIGARCH(1,d,1) d of ARFIMA = -0.0998, d of FIGARCH = 0.5277* (0.6300) (0.0000)	9.877	53.306
ARFIMA(3,d,3)-FIGARCH(1,d,1) d of ARFIMA = -0.0491, d of FIGARCH = 0.5568* (0.5356) (0.0000)	11.836	59.226

ARFIMA-FIAPARCH	AIC	SBIC
ARFIMA(1,d,1)-FIAPARCH(1,d,1) d of ARFIMA = -0.0901, d of FIAPARCH = 0.3786* (0.3152) (0.0001)	5.848	41.381
ARFIMA(1,d,2)-FIAPARCH(1,d,1) d of ARFIMA = -0.0858, d of FIAPARCH = 0.3797* (0.5508) (0.0002)	7.848	47.329
ARFIMA(2,d,3)-FIAPARCH(1,d,1) d of ARFIMA = -0.1388, d of FIAPARCH = 0.3694** (0.7469) (0.0203)	11.843	59.219
ARFIMA(3,d,3)-FIAPARCH(1,d,1) d of ARFIMA = -0.0475, d of FIAPARCH = 0.3940* (0.6683) (0.0005)	13.823	65.147

Note: * and ** significant at the 1% and 5% level, respectively.

Table 4: MAE and MAPE of rTVIX by ARFIMA-FIGARCH and ARFIMA-FIAPARCH

ARFIMA-FIGARCH

Model	Day	1	2	3	4	5	Average
ARFIMA(1,d,1)-FIGARCH(1,d,1)	MAE	0.02	0.23	0.23	0.08	0.19	0.15
	MAPE (%)	1.67	0.95	0.99	0.94	1.00	1.11
ARFIMA(1,d,2)-FIGARCH(1,d,1)	MAE	0.01	0.24	0.23	0.08	0.19	0.15
	MAPE (%)	0.69	1.00	1.00	0.98	0.99	0.93
ARFIMA(1,d,3)-FIGARCH(1,d,1)	MAE	0.02	0.23	0.23	0.08	0.20	0.15
	MAPE (%)	1.58	0.95	0.98	0.95	1.01	1.09
ARFIMA(2,d,3)-FIGARCH(1,d,1)	MAE	0.02	0.24	0.23	0.08	0.20	0.15
	MAPE (%)	1.63	0.96	0.98	0.93	1.01	1.10
ARFIMA(3,d,3)-FIGARCH(1,d,1)	MAE	0.00	0.25	0.21	0.08	0.18	0.14
	MAPE (%)	0.22	1.01	0.90	0.98	0.91	0.80

ARFIMA-APARCH

Model	Day	1	2	3	4	5	Average
ARFIMA(1,d,1)-FIAPARCH(1,d,1)	MAE	0.02	0.23	0.23	0.08	0.19	0.15
	MAPE (%)	1.56	0.96	0.99	0.94	1.00	1.09
ARFIMA(1,d,2)-FIAPARCH(1,d,1)	MAE	0.02	0.23	0.23	0.08	0.19	0.15
	MAPE (%)	1.56	0.96	0.99	0.93	1.00	1.09
ARFIMA(2,d,3)-FIAPARCH(1,d,1)	MAE	0.02	0.23	0.23	0.08	0.19	0.15
	MAPE (%)	1.40	0.96	0.99	0.93	1.00	1.06
ARFIMA(3,d,3)-FIAPARCH(1,d,1)	MAE	0.00	0.24	0.21	0.08	0.18	0.15
	MAPE (%)	0.34	1.00	0.92	1.01	0.93	0.84

Figure 2.1: Actual and Forecasting rTVIX by ARFIMA(1,d,1)-FIGARCH(1,d,1)

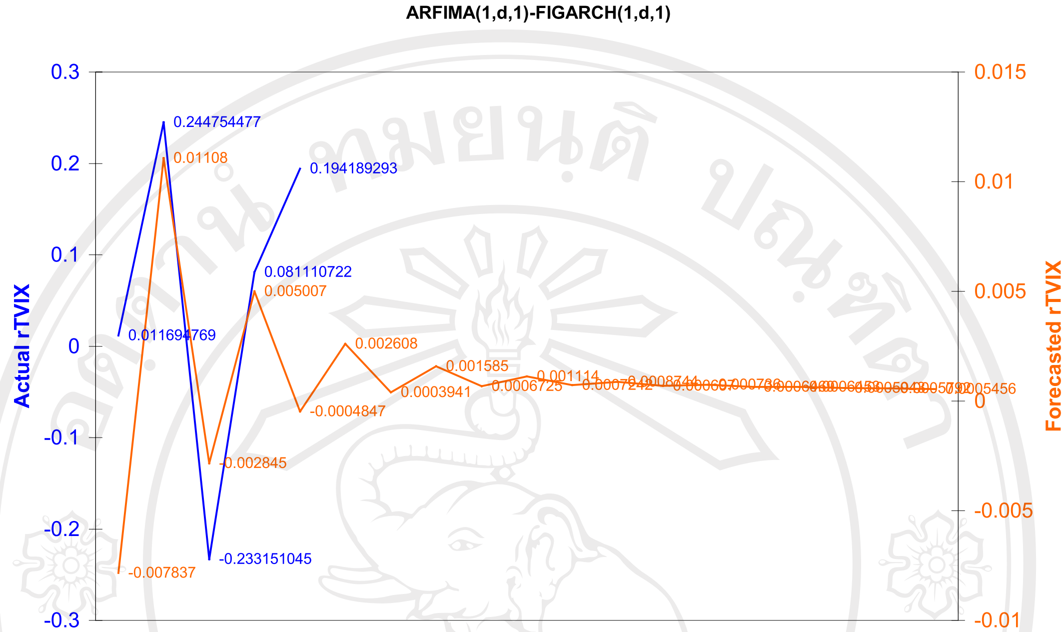

Figure 2.2: Actual and Forecasting rTVIX by ARFIMA(1,d,2)-FIGARCH(1,d,1)

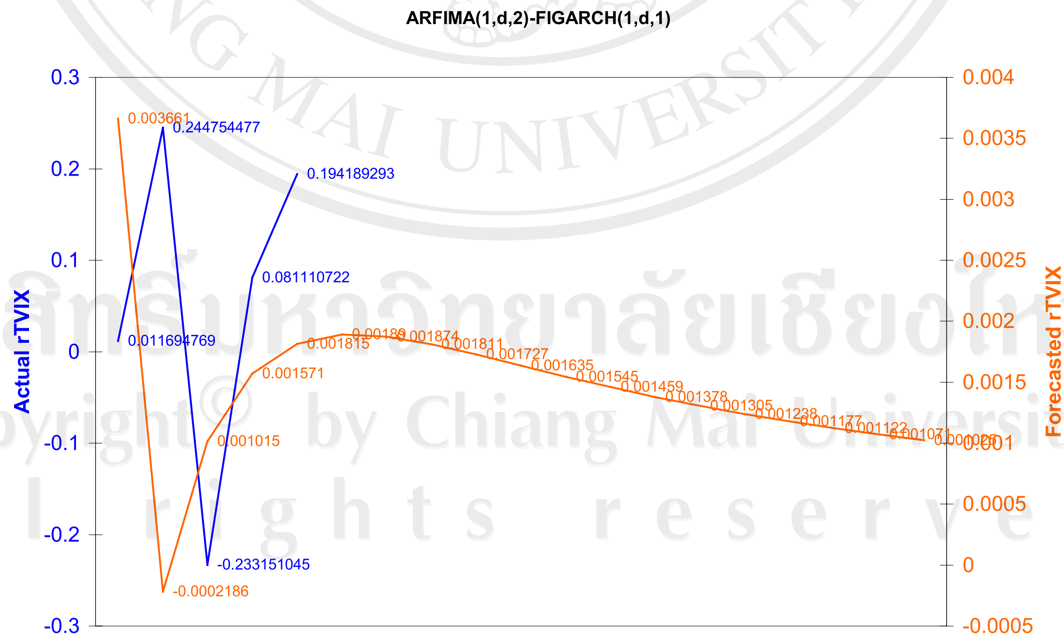

Figure 2.3: Actual and Forecasting rTVIX by ARFIMA(1,d,3)-FIGARCH(1,d,1)

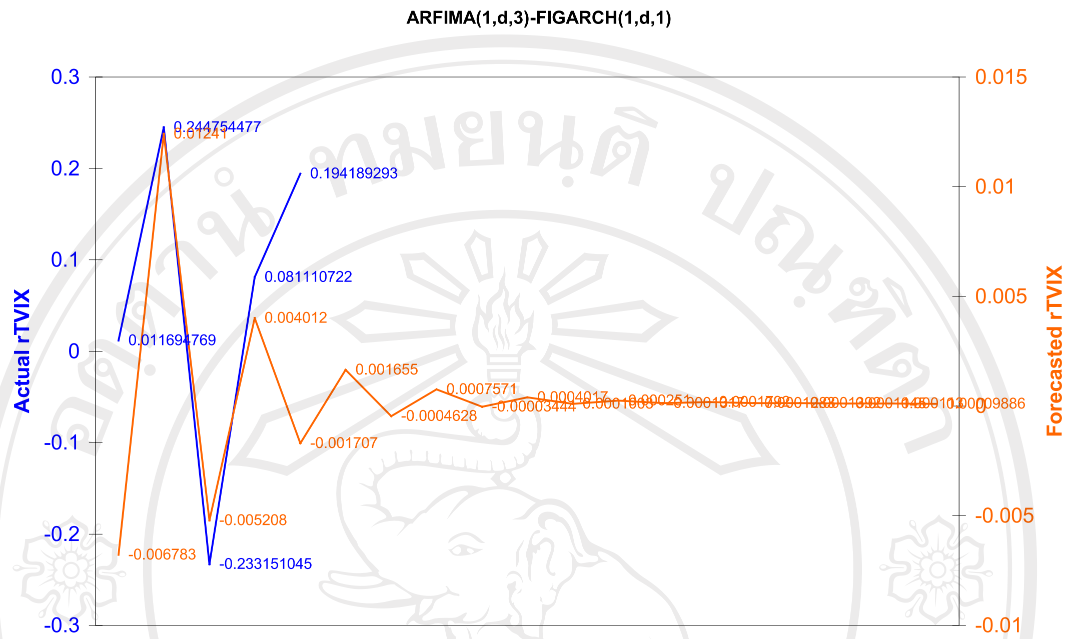

Figure 2.4: Actual and Forecasting rTVIX by ARFIMA(2,d,3)-FIGARCH(1,d,1)

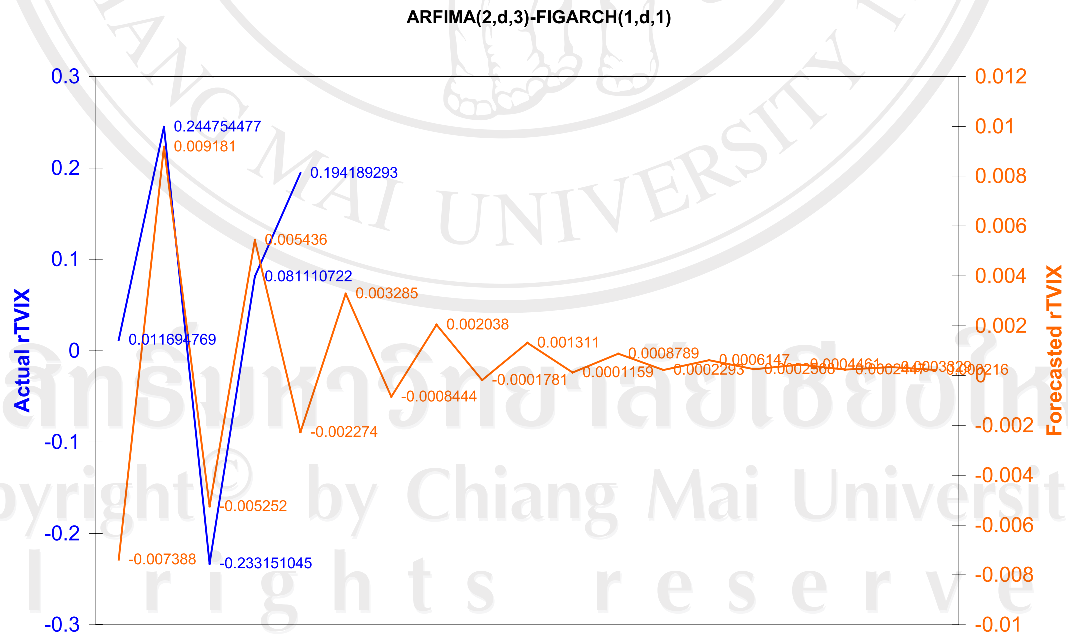

Figure 2.5: Actual and Forecasting rTVIX by ARFIMA(3,d,3)-FIGARCH(1,d,1)

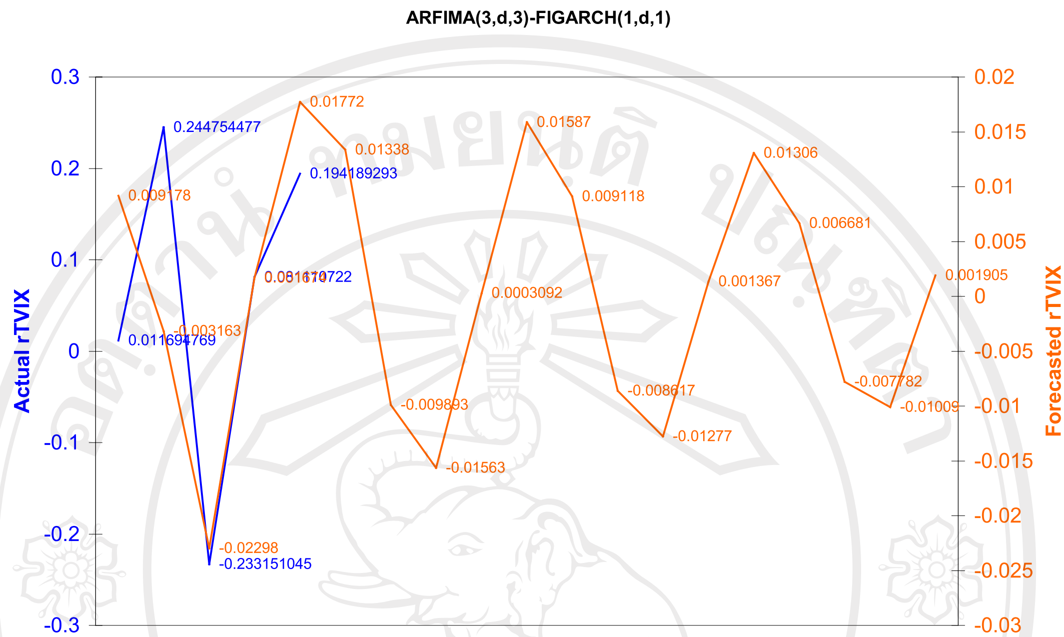

Figure 3.1: Actual and Forecasting rTVIX by ARFIMA(1,d,1)-FIAPARCH(1,d,1)

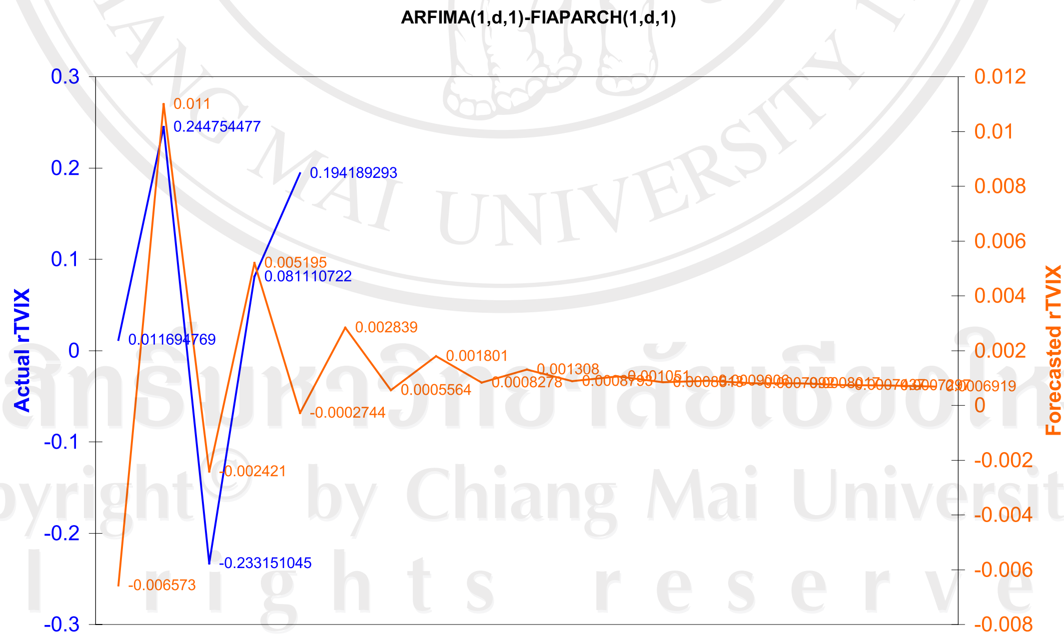

Figure 3.2: Actual and Forecasting rTVIX by ARFIMA(1,d,2)-FIAPARCH(1,d,1)

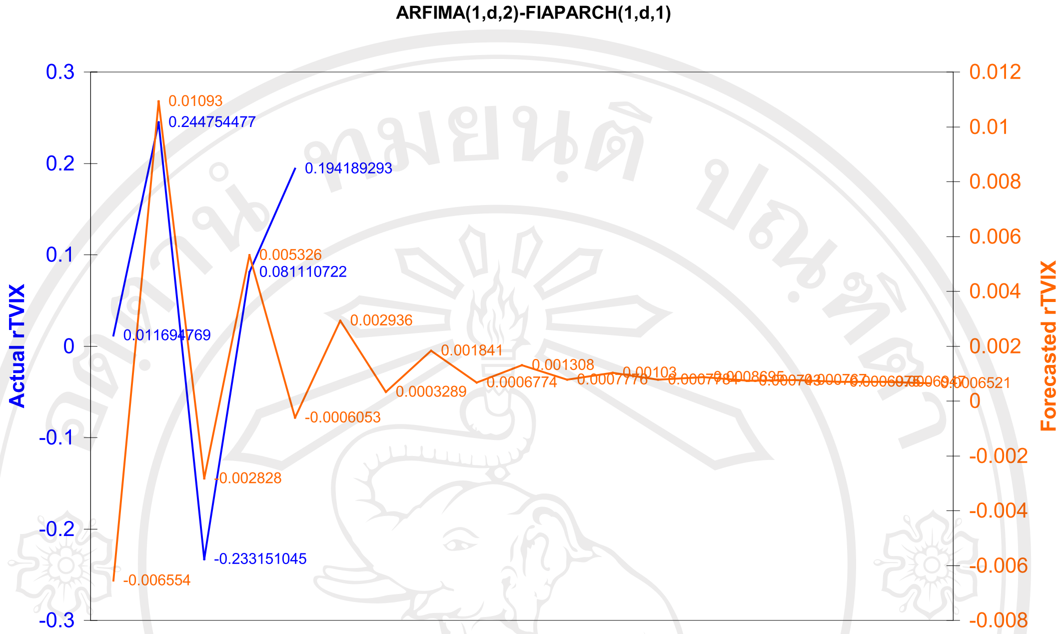

Figure 3.3: Actual and Forecasting rTVIX by ARFIMA(2,d,3)-FIAPARCH(1,d,1)

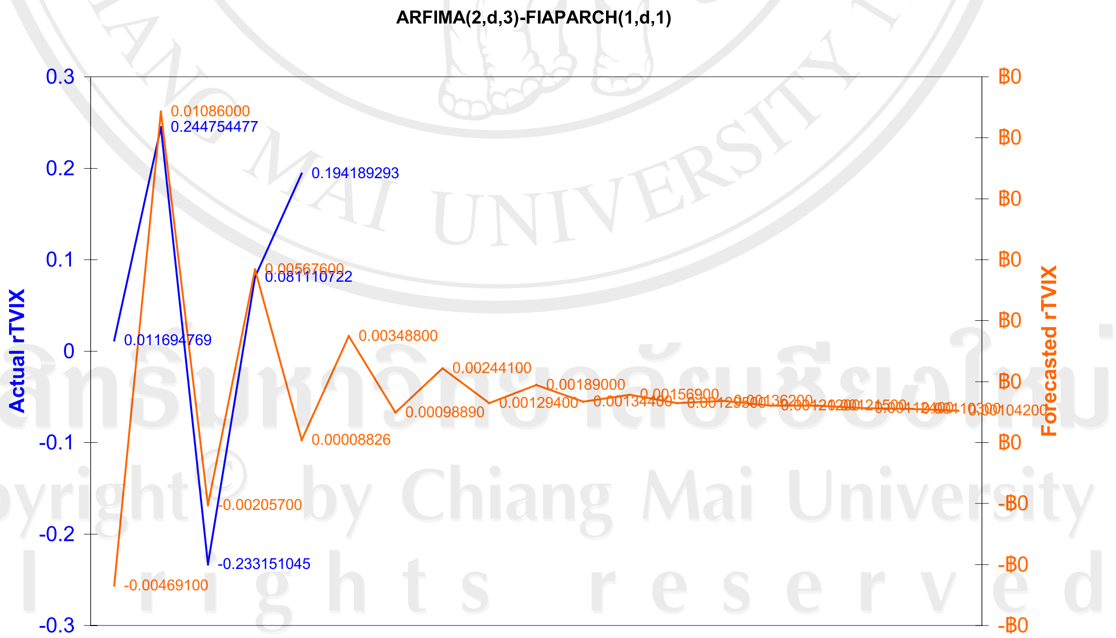

Figure 3.4: Actual and Forecasting rTVIX by ARFIMA(3,d,3)-FIAPARCH(1,d,1)

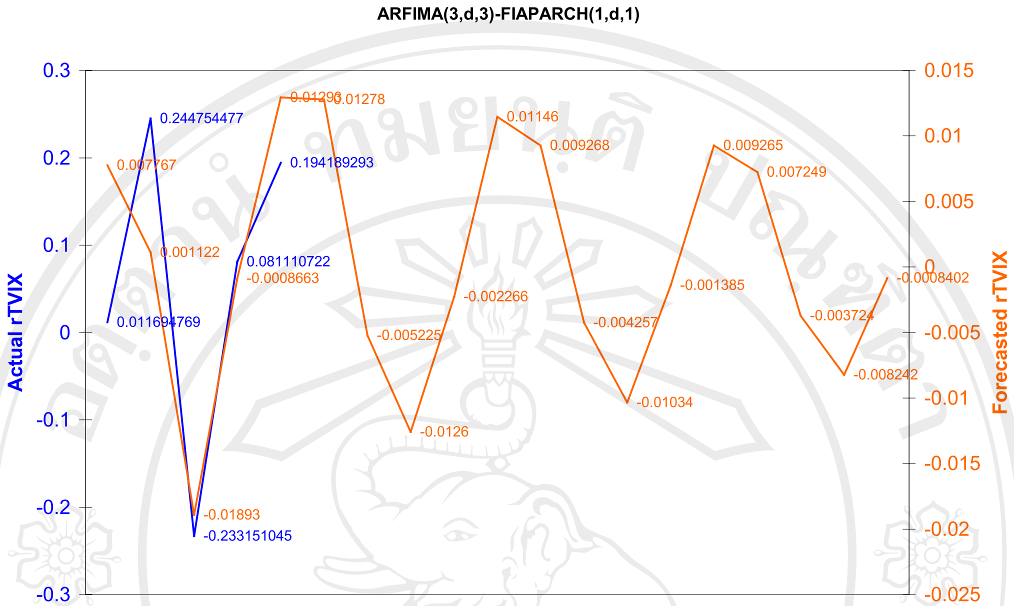

Table 5: ARMA-FIGARCH and -FIAPARCH on Returns of TVIX

	ARMA-FIGARCH		ARMA-FIAPARCH	
	Coefficient	**t-value**	**Coefficient**	**t-value**
AR(1)	0.7806*	6.0140 (0.000)	-0.5026	-0.9686 (0.3334)
MA(1)	-1.0190*	-6.2510 (0.000)	0.2924	0.4962 (0.6200)
MA(2)	0.1750*	2.7090 (0.0071)	-	- -
ω	0.0241	0.9844 (0.3256)	0.0907	0.9409 (0.3473)
α	-0.5326*	-2.8270 (0.005)	-0.4586	-0.9125 (0.3621)
β	-0.3833	-1.6130 (0.1076)	-0.3869	-0.6275 (0.5307)
γ	-	- -	-0.3267	-1.1520 (0.2500)
δ	-	- -	1.3510*	3.498 (0.0005)
D	0.4026*	5.3820 (0.000)	0.5457*	3.7250 (0.0002)
AIC	1.886		3.856	
SBIC	29.522		35.440	

Note: * Significant at the 1% level

APPENDIX C

The Comparison among ARMA-GARCH, -EGARCH, -GJR, and -PGARCH models on Thailand Volatility Index

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The Comparison among ARMA-GARCH, -EGARCH, -GJR, and -PGARCH models on Thailand Volatility Index*

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ABSTRACT

With the formulae of Volatility Index (VIX) which was launched by the Chicago Board Options Exchange (CBOE) in 2003, SET50 Index options is applied as a Thailand Volatility Index (TVIX). We estimate ARMA-GARCH, -EGARCH, -GJR and -PGARCH models for Thailand Volatility Index (TVIX). These models are the extension of ARCH process with various features to explain the obvious characteristics of financial time series such as asymmetric and leverage effect. As we apply TVIX with these models, the comparison and forecast are performed.

4. Introduction

In recent years, financial crises impact global economy. The crises dramatically cause recession in commodities and money markets because of the liquidity shrinking. While the decrease in most assets occurs, an important figure in financial market, called volatility index, inversely turn. The price with high volatility reflects higher risk in holding such asset. The volatility can be calculated in a numerical value as an index known as Volatility Index.

In 1993, the Chicago Board Options Exchange (CBOE) introduced the CBOE Volatility Index, VIX, which quickly became the benchmark for stock market volatility. In 2003, the CBOE made two key enhancements to the VIX methodology.

The new VIX is based on an up-to-the-minute market estimation of expected volatility that is calculated by using real-time S&P500 Index (SPX) option bid/ask quotes. Until 2006, VIX was trading on the CBOE. The VIX options contract is the first product on market volatility to be listed on an SEC-regulated securities exchange. This new product can be traded from an options-approved securities account. Many investors consider the VIX to be the world's premier barometer of investor sentiment and market volatility, and VIX options are a very powerful risk management tool.

The early generation of GARCH models, such as the ARCH and

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GARCH models have the ability of reproducing another very important stylized fact, which is volatility clustering; that is, big shocks are followed by big shocks. However, only the magnitude of the shock, but not the sign, affects conditional volatility. Therefore, the first generation of GARCH models cannot capture the stylized fact that bad (good) news increase (decrease) volatility. This limitation has been overcome by the introduction of more flexible volatility specifications which allow positive and negative shocks to have a different impact on volatility. This more recent class of GARCH models includes the Exponential GARCH (EGARCH), the Glosten, Jagannathan, and Runkle-GARCH (GJR-GARCH) and the Power GARCH (PGARCH) model. Finally, a new class of GARCH models which jointly capture leverage effects and contemporaneous asymmetry, as well as time varying skewness and kurtosis, has been recently introduced by El Babsiri and Zakoian (2001). In a recent paper, Patton (2004) also analyzes the use of asymmetric dependence among stocks; that is, the fact that stocks are more highly correlated during market downturns.

In this paper, we applied VIX and compare the conditional variance among various GARCH models which are GARCH, EGARCH, GJR-GARCH and PGARCH models. Nevertheless, it should be pointed out that several empirical studies have already examined the impact of asymmetries on the forecast performance of GARCH models. The recent survey by Poon and Granger (2003) provides, among other things, an interesting and extensive synopsis of them. Indeed, different conclusions have been drawn from these studies. In fact, some studies find evidence in favor of asymmetric models, such as EGARCH, for the case of exchange rates and stock returns predictions. Examples include Cao and Tsay (1992), Heynen and Kat (1994), Lee (1991), and Pagan and Schwert (1990). Other studies find evidence in favor of the GJR-GARCH model. The studies of Taylor (2001) also examine the case of stock returns volatility, and Bali (2000) for interest rate volatility. For PGARCH, interesting evident can be found from the study of Sebastien Laurent which derives analytical expressions for the score of the PGARCH model of Ding, Granger, and Engle (1993).

The rest of the paper is organized as follows: Section 2 presents the CBOE VIX formula which the adaptation of the VIX to Thailand SET50 Index options, the Thailand Volatility Index (TVIX), can be estimated. Section 3 formally defines theory and process of GARCH, EGARCH, GJR-GARCH, and PGARCH models. The data is shown in section 4 which daily returns of TVIX are described. The estimation of ARMA-GARCH, -EGARCH, -GJR, and -PGARCH models are shown in the final section. This section provides tables and figures of family of GARCH on Returns of TVIX and the comparison of test statistics, together with a brief conclusion.

2. Volatility Index

Estimating implied volatility from options is no straightforward method to extract the information. Whaley (2000) considered implied volatility as a fear gauge because option prices calculate implied volatility that represents a market-based estimate of future price volatility). Implied volatilities are the information by investors, financial news services and other finance professionals. The information content and forecast quality of implied volatility is an important topic in financial markets research.

Latane and Rendleman (1976), Chiras and Manaster (1978), Beckers (1981) and Jorion (1995) provided early assessments of the forecast quality of implied volatility and concluded that implied volatility outperforms historical standard deviations and is a good predictor of future volatility, although it might be biased. Christensen and Prabhala (1998) also found that implied volatility forecasts are biased, but dominate historical volatility in terms of ex ante forecasting power. Fleming (1998) used a historical volatility measure to show that implied volatilities outperform historical information.

Dennis *et al.* (2006) found that daily innovations in VIX contain very reliable incremental information about the future volatility of the S&P100 index. Other studies that attempt to forecast implied volatility or use the information contained in implied volatility to trade in option markets include Harvey and Whaley (1992), Noh *et al.* (1994), and Poon and Pope (2000).

The New VIX is more robust because it pools the information from option prices over the whole volatility skew, and not just from at-the-money options. The formula used in the new VIX calculation is given by the CBOE as follows:

$$\sigma^2 = \frac{2}{T}\sum_{i}\frac{\Delta K_i}{K_i^2}e^{RT}Q(K_i) - \frac{1}{T}\left[\frac{F}{K_0}-1\right]^2$$

where

σ = VIX / 100 (so that VIX = σ x 100),

T = Time to expiration (in minutes),

F = Forward index level, derived from index option prices (based on at-the-money option prices, the difference between call and put prices is smallest).

The formula used to calculate the forward index level is:

F = Strike price (at-the-money) + e^{RT} x (Call price – Put price),

where

R = risk-free interest rate is assumed to be 3.01% (for simplicity, the government T-bills 3 month contract interest rate is used, as the Thailand options contract is a 3 months contract);

T = $\{M_{current\ day} + M_{settlement\ day} + M_{other\ days}\}$/minutes in a year,

where

$M_{current\ day}$ = # of minutes remaining until midnight of the current day,

$M_{settlement\ day}$ = # of minutes from midnight until 9:45 am on the TFEX settlement day,

$M_{other\ days}$ = Total # of minutes in the days between the current day and the settlement day;

K_i = Strike price of i^{th} out-of-the-money option; a call if $K_i > F$ and a put if $K_i < F$;

ΔK_i = Interval between strike prices - half the distance between the strike on either side of K_i: $\Delta K_i = \frac{K_{i+1} - K_{i-1}}{2}$.

K_0 = First strike below the forward index level, F;

$Q(K_i)$ = The midpoint of the bid-ask spread for each option with strike K_i.

(Note: ΔK_i for the lowest strike is simply the difference between the lowest strike and the next higher strike. Likewise, ΔK for the highest strike is the difference between the highest strike and the next lower strike.)

With the adaptation of the VIX calculation to Thailand SET 50 index options, the Thailand expected volatility (TVIX) can be estimated.

3. Theory

3.1 GARCH Model

GARCH model by Bollerslev (1986) imposes important limitations, not to capture a positive or negative sign of u_t, which both positive and negative shocks has the same impact on the conditional variance, h_t, as follows,

$$u_t = \eta_t \sqrt{\sigma_t},$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i u_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2$$

where $\omega > 0$, $\alpha_i \geq 0$ for i = 1,…,p and $\beta_j \geq 0$ for j = 1,…,q are sufficient to ensure that the conditional variance, σ_t, is non-negative. For the GARCH process to be defined, it is required that $\omega > 0$. Also, a univariate GARCH(1,1) model is known as ARCH(∞) model (Engle, 1982) as an infinite expansion in u_{t-i}^2. The α represents the ARCH effect and β represents the GARCH effect.

3.2 Exponential GARCH (EGARCH) Model

Exponential GARCH (EGARCH) by Nelson (1991) is the logarithm of conditional volatility in order to capture asymmetries between positive and negative shocks that the leverage effect is exponential, as follows,

$$\log(\sigma_t^2) = \omega + \sum_{i=1}^{p} \alpha_i \left|\eta_{t-i}\right| + \sum_{j=1}^{q} \beta_j \log\left(\sigma_{t-j}^2\right) + \sum_{k=1}^{r} \gamma_k \left|\eta_{t-k}\right|$$

where $\eta_{t-i} = \frac{u_{t-i}}{\sigma_{t-i}}$ which η_{t-i} and $\left|\eta_{t-i}\right|$ capture the sign and size effects of the standardized shocks. There are no restrictions on the parameters in the model. The moment conditions of the model are also straightforward because the standardized shocks have finite moments. There is an leverage effect when $\gamma < \alpha < -\gamma$. This implies that the negative shocks increase volatility and vice versa.

3.3 Glosten, Jagannathan and Runkle (GJR-GARCH) Model

Glosten, Jagannathan and Runkle (GJR-GARCH) model by Glosten *et al.* (1993) is to capture possible asymmetric impacts of positive and negative shocks on the conditional variance for σ_t, as follows,

$$\sigma_t = \omega + \sum_{i=1}^{p} (\alpha_i + \gamma_i I(u_{t-i})) u_{t-j}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j},$$

where $I(u_{t-j})$ is an indicator function that equals to 1 if $u_{t-j} < 0$ and 0 otherwise. If p = q = 1, $\omega > 0, \alpha_1 \geq 0, \alpha_1 + \gamma_1 \geq 0$ and $\beta_1 \geq 0$ are sufficient conditions to ensure that the conditional variance h_t is non-negative. $\alpha_1(\alpha_1 + \gamma_1)$ gives the short-run persistence of positive (negative) shocks. If $\gamma_1 \neq 0$, the news impact is asymmetry. If $\gamma > 0$, there is a leverage effect that bad news increases volatility.

Lee and Hansen (1994) derived the log-moment condition for GARCH(1,1) of conditional volatility which is sufficient for the statistical properties of the QMLE to be consistent, as follows,

$$E(\log(\alpha_1 \eta_t^2 + \beta_1)) < 0.$$

It is essential to note that the log-moment condition is a weaker regularity condition than the second moment condition. Therefore, the second moment is sufficient condition for consistency and asymptotic normality of the QMLE, as follows,

$$\alpha_1 + \beta_1 < 1.$$

Moreover, McAleer *et al.* (2007) established the log-moment and second moment condition for GJR(1,1) as follows,

$$E(\log((\alpha_1 + \gamma_1 I(\eta_t))\eta_t^2 + \beta_1)) < 0$$
$$\text{and } \alpha + (\gamma / 2) + \beta < 1$$

Both moments are the sufficient conditions for the consistency and asymptotic normality of the QMLE for the GJR(1,1).

3.4 The Power GARCH (PGARCH) Model

The Power GARCH (PGARCH) model by Taylor (1986) and Schwert (1989) use the conditional standard deviation as a measure of volatility instead of the conditional variance. This model is generalized by Ding *et al.* (1993) using the PGARCH model as follows :

$$\sigma_t^{\delta} = \omega + \sum_{i=1}^{q} \alpha_i \left(|u_{t-i}| - \gamma_i u_{t-i} \right)^{\delta} + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^{\delta}$$

where $\delta > 0, |\gamma_i| \leq 1 \, for \, i = 1,2,...,r \, an$
$d \, \gamma_i = 0 \, for \, i > r, and \, r \leq p.$

In the PGARCH model, if $\gamma \neq 0$, this captures asymmetric effects. The PGARCH model reduces to the GARCH model when $\delta = 2$ and $\gamma_i = 0$ for all *i*.

4. Data Description

One-minute intervals of SET50 Index options are obtained from Bloomberg, accounted by the Faculty of Economics, Chiang Mai University and Research Institute, Stock Exchange of Thailand. The sample period is from 27 January 2008 until 30 September 2009. The contract months are March, June, September 2008 and 2009 and December 2008.

In order to calculate TVIX, we utilize the SAS 9.1 software package for the calculation as it offers a number of features that are not available in traditional econometric software. For the estimation, we use daily returns of TVIX to estimate ARMA-GARCH, -EGARCH, -GJR-GARCH, and -PGARCH by using E-Views 6.0 software.

The returns of TVIX at time t are calculated as follows:

$$R_{i,t} = \log(P_{i,t} / P_{i,t-1})$$

where $P_{i,t}$ and $P_{i,t-1}$ are the closing prices of TVIX at time t and $t-1$, respectively.

[Insert Table 1 around here]

Table 1 presents the descriptive statistics for the returns of TVIX. The average return of TVIX is negative. The normal distribution has a skewness statistic equal to zero and a kurtosis statistic of 3, but return of TVIX has negative skewness statistics and high kurtosis, suggesting the presence of fat tails and a non symmetric series. This means that the data has a longer left tail (extreme losses) than right tail (extreme gain). The relatively large kurtosis indicates non-normality that the distribution of returns is leptokurtic. This suggests that the market shocks of either sign for the TVIX returns are more likely to be observed. Jarque-Bera normality test rejects the hypothesis of normality for the sample.

Figure 1 presents the plot of TVIX and TVIX returns. This indicates some circumstances where TVIX returns fluctuate. Table 2 summarized the unit root tests for TVIX returns. The Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests were used to test the null hypothesis of a unit root against the alternative hypothesis of stationarity. The tests yield large negative values in all cases for levels such that the individual returns series reject the null hypothesis at the 1% significance level, hence, the returns are stationary.

[Insert Figure 1 around here]
[Insert Table 2 around here]

5. Estimation

[Insert Table 3 around here]

Table 3 represents the ARCH and GARCH effects from statistically significant at 5% level of α and β. It shows that the long-run coefficients are all statistically significant in the variance equation. The coefficients of α appears to show the presence of volatility clustering in the models. Conditional volatility for the models tends to rise (fall) when the absolute value of the standardized residuals is larger (smaller). The coefficients of β (a determinant of the degree of persistence) for all models are less than 1 showing persistent volatility.

However, the coefficients of γ, the asymmetry and leverage effects, are negative and statistically significant at the 1% level in the GJR-GARCH and PGARCH models and positive and statistically significant at the 1% level in the EGARCH model. However, the leverage effect only exists if $\gamma < 0$ in the EGARCH model and $\gamma > 0$ in the GJR-GARCH and PGARCH models. This appears that there is asymmetric in all models as $\gamma \neq 0$ but the hypothesis of leverage effect is rejected for all models.

[Insert Table 4 around here]

For GARCH(1,1) and GJR(1,1), the results are shown on Table 4. The second moment condition is only calculated, and it can be used to verify consistency and asymptotic normality of QMLE in the event that the log-moment condition cannot be computed because $((\alpha_1 + \gamma_1 I(\eta_t))\eta_t^2 + \beta_1))$ less than zero for any t = 1, 2, …, n (McAleer et al. (2009)).

The second moment condition shows the satisfaction rate, the value of which is less than unity in all cases. Hence, the consistency and asymptotic normality of the QMLE are guaranteed.

[Insert Table 5 around here]

In terms of the lowest AIC criteria, the best model is the PGARCH model but in terms of the lowest SBIC, the best model is the EGARCH model. From table 5, the ARMA-GJR has the lowest MAPE and RMSE. In addition, GJR-GARCH model is satisfied by the second moment that is a sufficient condition for the consistency and asymptotic normality of the QMLE. Therefore, GJR-GARCH is the best model.

6. Conclusion

This paper calculates the Thailand volatility index (TVIX) by applying CBOE Volatility Index (VIX) and SET50 Index options data, and estimates the volatility of TVIX returns using ARMA-GARCH, -EGARCH, -GJR-GARCH, and -PGARCH models. Volatility persistence and asymmetric properties are analyzed.

The results from all of the models show the volatility with statistically significant asymmetry effect with all the models but without leverage effects. This is in contrast to the work of Nelson (1991). The ARMA-PGARCH is found to be the best model with the lowest AIC criteria values but the EGARCH model has the lowest SBIC criteria value. Regarding MAPE and RMSE criteria, GJR-GARCH is the best fitting model for TVIX.

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Table 1: Descriptive Statistics of TVIX Returns

	Mean	Std Dev	Skewness	Kurtosis	Max	Min
TVIX	-0.00022	0.093806	-1.08530	11.41235	0.44728	-0.52266

Figure 1: Daily TVIX and Returns

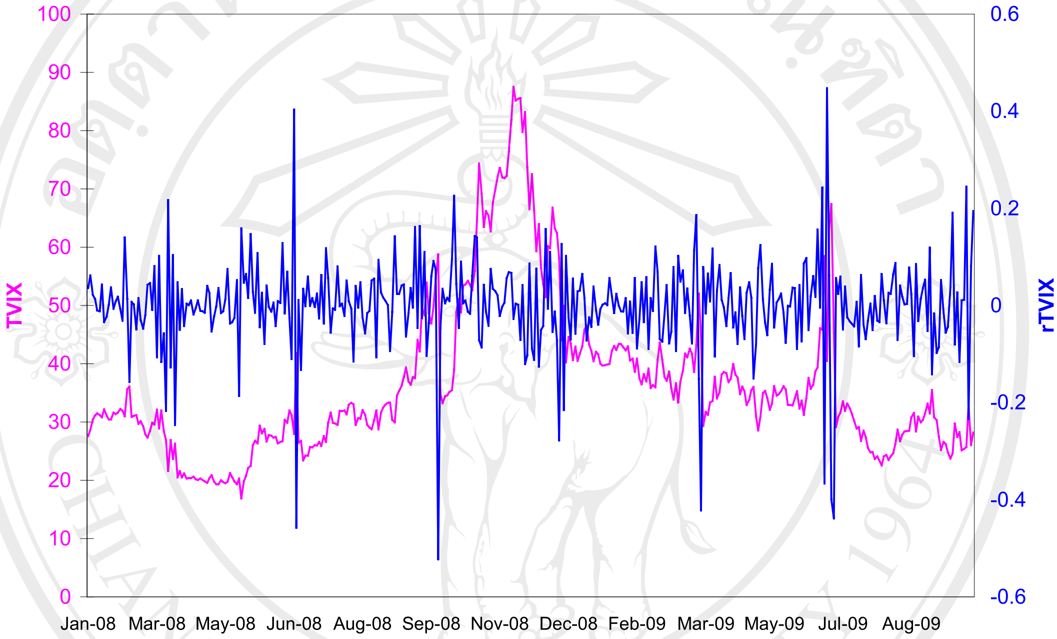

Table 2: Unit Root Test for Returns of TVIX

	ADF Test			Phillips-Perron Test		
Returns	None	Constant	Constant and Trend	None	Constant	Constant and Trend
TVIX	-24.022*	-23.991*	-23.977*	-24.560*	-24.526*	-24.522*

Note: * significant at the 1% level.

Table 3: Family of GARCH on Returns of TVIX

	Mean Equation			Variance Equation			
	Coefficient	**z-Statistic**		**Coefficient**	**z-Statistic**	**AIC**	**SBIC**
GARCH							
Constant (Mean)	-0.0000	-0.0359 (-0.9714)	ω	0.0004*	5.4432 (0.0000)	-3.899	-3.828
AR(1)	-0.7184**	-8.3029 (0.000)	α	0.7149*	7.7283 (0.0000)		
MA(1)	0.5647**	4.8369 (0.000)	β	0.2441*	3.5503 (0.0000)		
MA(31)	-0.0772**	-2.4124 (0.0158)					
E-GARCH							
Constant (Mean)	0.0016	1.1975 (0.2311)	ω	0.3676*	-5.2149 (0.0000)	-3.921	-3.839
AR(1)	-0.7025**	-6.9355 (0.000)	α	0.0826*	8.3123 (0.0000)		
MA(1)	0.5432**	4.1779 (0.0000)	β	0.7896*	2.6898 (0.0072)		
MA(31)	-0.0717**	-2.2250 (0.0261)	γ	0.0579*	15.9725 (0.0000)		
GJR-GARCH							
Constant (Mean)	0.0014	1.0001 (0.3173)	ω	0.0004*	5.5956 (0.0000)	-3.910	-3.828
AR(1)	-0.7149**	-8.1660 (0.0000)	α	1.0561*	4.9896 (0.0000)		
MA(1)	0.5670**	4.7735 (0.0000)	β	0.2525*	3.4095 (0.0007)		
MA(31)	-0.0749**	-2.4047 (0.0162)	γ	-0.6393*	-2.8496 (0.0044)		
PGARCH							
Constant (Mean)	0.0017	1.4412 (0.1495)	ω	0.0134	1.1377 (0.2553)	-3.925	-3.833
AR(1)	-0.7115*	-8.2320 (0.0000)	α	0.4307*	6.5486 (0.0000)		
MA(1)	0.5603*	4.9346 (0.0000)	β	0.4526*	5.6521 (0.0000)		
MA(31)	-0.0804*	-2.7512 (0.0059)	γ	-0.2951*	-3.5959 (0.0003)		
			δ	0.8738*	3.7207 (0.0002)		

Note: * Significant at the 1% level
** Significant at the 5% level

Table 4: Second moment condition for ARMA-GARCH and ARMA-GJR

ARMA-GARCH	ARMA-GJR
0.9590	0.9890

Table 5: Comparison of test statistics for family of GARCH

	MAPE	RMSE
ARMA-GARCH	134.9544	0.093361
ARMA-EGARCH	133.0396	0.093453
ARMA-GJR	132.4593	0.093450
ARMA-PGARCH	134.3143	0.093484

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